B.TECH/ME/4TH SEM/MATH 2001 (BACKLOG)/2022

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The period of the function $f(x) = |\sin x|$ is
(a) 2π (b) $\frac{\pi}{2}$ (c) 3π (d) π

(ii) If $F(s)$ be the Fourier transform of $f(x)$, then the Fourier transform of $f(x+a)$ is
(a) $e^{is}F(s)$ (b) $F(s)$ (c) $e^{ias}F(s)$ (d) $e^{-ias}F(s)$

(iii) If $f(z) = \frac{\sin z}{z^4-2z^3}$, then $z=0$ is pole of order
(a) 1 (b) 2 (c) 3 (d) 4

(iv) One dimensional wave equation is of the form
(a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ (b) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
(c) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$ (d) $\frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x}$
where $u(x,t)$ is a function of x and t and c is a constant.

(v) The function $f(z) = \frac{e^{z^2}}{z^4}$ has
(a) essential singularity at $z=0$ (b) a pole of order 4 at $z=0$
(c) a simple pole at $z=0$ (d) removable singularity at $z=0$

(vi) If $f(x) = sinx$, $-\pi \le x < \pi$ be expanded in Fourier Series as $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ then a_0 is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(vii) The solution of the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$ is
(a) $P_0(x)$ (b) $P_1(x)$ (c) $P_3(x)$ (d) $P_4(x)$

(viii) For the differential equation $3xy'' + 2x(x-1)y' + 5y = 0$,
(a) $x = 0$ is the regular singular point
(b) $x = 0$ is the irregular singular point
(c) $x = 0$ is the ordinary point
(d) no singular point exist.

(ix) The solution of $xp + yq = z$ is
(a) $f(x, y)$ (b) $f(\frac{x}{y}, \frac{y}{z})$ (c) $f(xy, yz)$ (d) $f(x^2, y^2)$

(x) For $f(z) = \frac{\sin z}{z}$, the point $z = 0$ is
(a) an essential singularity (b) a non-isolated singularity
(c) a removable singularity (d) a pole of order 2.

Group- B

2. (a) Prove that the function defined by

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is not analytic at the origin although Cauchy-Riemann equations are satisfied at the origin. [(MATH2001.1, MATH2001.2)(Understand/LOCQ)]

(b) Evaluate $\oint_C \frac{\cos^3 z}{\left(z-\frac{\pi}{4}\right)^3} dz$ where C is the circle $|z| = 1$.

[(MATH2001.1, MATH2001.2)(Apply/IOCQ)]

6 + 6 = 12

3. (a) Find the value of the integral $\int_0^{1+i} (x - y + ix^2)\, dz$ along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $1 + i$.
[(MATH2001.1, MATH2001.2)(Evaluate/HOCQ)]

(b) Find the Laurent's series expansion of $f(z) = \frac{z}{(z-1)(z-2)}$ about $z = -2$ in all possible regions. [(MATH2001.1, MATH2001.2)(Apply/IOCQ)]

6 + 6 = 12

Group - C

4. (a) Find Fourier series expansion of $f(x) = x^2$ in $-\pi < x < \pi$, and hence show that $\sum \frac{1}{n^4} = \frac{\pi^4}{90}$ by using Parseval's identity.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Apply/IOCQ)]

(b) Find $F^{-1}\left(\frac{1}{s^2+4s+13}\right)$.
[(MATH2001.1, MATH2001.3, MATH2001.4)(Remember/LOCQ)]

7 + 5 = 12

5. (a) Obtain the Fourier series for the function $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi. \end{cases}$$

[(MATH2001.1, MATH2001.3, MATH2001.4)(Apply/IOCQ)]

(b) Find f(x) if its Fourier cosine transform is $F_c(s) = \begin{cases} a - \frac{s}{2}, \text{for } s < 2a \\ 0 \quad, \text{for } s \ge 2a \end{cases}$.

[(MATH2001.1, MATH2001.3, MATH2001.4)(Evaluate/HOCQ)]

6 + 6 = 12

Group - D

6. (a) Find the power series of $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$ about $x = 0$.

[(MATH2001.5)(Evaluate/HOCQ)]

(b) Prove that $nP_n = xP_n{}' - P_{n-1}{}'$ where $P_n(x)$ is Legendre's polynomial of order n.

[(MATH2001.5)(Remember/LOCQ)]

8 + 4 = 12

7. (a) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$, where $J_n(x)$ is the Bessel's function of first kind.

[(MATH2001.5)(Apply/IOCQ)]

(b) Find the value of $P_n(1)$. Hence prove that $P_n'(1) = \frac{1}{2}n(n+1)$.

[(MATH2001.5)(Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) Form the partial differential equation by eliminating the arbitrary function 'f' from

$$f(xy + z^2,\ x + y + z) = 0.$$

[(MATH2001.1, MATH2001.6)(Create/HOCQ)]

(b) Find the general solution of the following partial differential equation
$(y + zx)p - (x + yz)q = x^2 - y^2$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]

6 + 6 = 12

9. (a) Solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$.

[(MATH2001.1,MATH2001.6)(Remember/LOCQ)]

(b) Solve the following partial differential equation using suitable method:

$$z^2 = pqxy.$$

[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	28.12	51.04	20.83

Course Outcome (CO):

After the completion of the course students will be able to

MATH2001.1 Construct appropriate mathematical models of physical systems.

MATH2001.2 Recognize the concepts of complex integration, Poles and Residuals in the stability analysis of engineering problems.

MATH2001.3 Generate the complex exponential Fourier series of a function and make out how the complex Fourier coefficients are related to the Fourier cosine and sine coefficients.

MATH2001.4 Interpret the nature of a physical phenomena when the domain is shifted by Fourier Transform e.g. continuous time signals and systems.

MATH2001.5 Develop computational understanding of second order differential equations with analytic coefficients along with Bessel and Legendre differential equations with their corresponding recurrence relations.

MATH2001.6 Master how partial differentials equations can serve as models for physical processes such as vibrations, heat transfer etc.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question