PHYSICS II
(PHYS 2211)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The temperature T is given by
(a) $\left(\frac{\partial U}{\partial S}\right)_P$ (b) $\left(\frac{\partial U}{\partial S}\right)_V$ (c) $\left(\frac{\partial S}{\partial U}\right)_P$ (d) $\left(\frac{\partial S}{\partial U}\right)_V$

(ii) The throttling process can be modelled by
(a) a reversible isentropic process (b) a reversible isenthalpic process
(c) a reversible isobaric process (d) an irreversible isenthalpic process

(iii) If the dead and alive states of the Schroedinger cat is given by $|A\rangle$ and $|D\rangle$ respectively
(a) $\langle A|D\rangle = 0$ (b) $\langle A|D\rangle = 1$ (c) $\langle A|D\rangle = \frac{1}{2}$ (d) $\langle A|D\rangle = \hbar$

(iv) The state of a quantum system is given by $|u\rangle = \frac{1}{\sqrt{5}}|u_1\rangle + \frac{2}{\sqrt{5}}|u_2\rangle$. The probability of finding the system in $|u_2\rangle$ is
(a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{1}{2}$ (d) $\frac{2}{\sqrt{5}}$

(v) The atomic radius of an bcc crystal (if a is lattice parameter) is
(a) a (b) $a\,/2$ (c) $a/(4/\sqrt{3})$ (d)) $a/(4/\sqrt{2})$

(vi) If $\vec{a}_i$ is primitive basis vector of a unit cell in direct lattice, and $\vec{b}_i$ is that in the corresponding reciprocal lattice then
(a) $\vec{a}_i.\vec{b}_j = o$ (b) $\frac{|\vec{a}_i|}{|\vec{b}_j|} = 2\pi\delta_{ij}$ (c) $\frac{|\vec{a}_i|}{|\vec{b}_j|} = e^{-i\,2\pi\delta_{ij}}$ (d) $\vec{a}_i.\vec{b}_j = 2\pi\delta_{ij}$

(vii) In an $(E\text{-}K)$ curve , the condition for energy discontinuity is
(a) $k = \pm\frac{n\pi}{a}$ (b) $k = \frac{2a}{n\pi}$ (c) $k = \pm\frac{a}{n\pi}$ (d) $k = \frac{n\pi}{a}$

(viii) An infinitesimal change in the Helmholtz free energy F is given by
(a) $dF = PdV + SdT$ (b) $dF = -PdV + SdT$
(c) $dF = PdV - SdT$ (d) $dF = -PdV - SdT$

(ix) Distance between two neighbouring atoms along [111] direction of an fcc crystal unit cell

(a) $a/\sqrt{3}$ (b) $a\sqrt{3}$ (c) $a/2$ (d) $a\sqrt{3}/2$

(x) The density of a charge carrier in a pure semiconductor is proportional to

(a) $e^{-\frac{E_g}{kT}}$ (b) $e^{-\frac{E_g}{kT^2}}$ (c) $e^{-\frac{2E_g}{kT}}$ (d) $e^{-\frac{E_g}{2kT}}$

Group - B

2\. (a) Suppose the equation of state of a thermodynamic system is given by $f(P,V,T)=0$ where the symbols have usual meanings. Show that

$$\left(\frac{\partial P}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_P\left(\frac{\partial T}{\partial P}\right)_V=-1$$

A certain system has an isothermal compressibility $\kappa=a/v$ and a volume expansivity $\beta=2bT/v$ where a and b are constants and v is the molar volume. Show that the equation of state of the system given by $v-bT^2+aP=constant$ satisfies the above thermodynamic requirement. [(CO2)(Remember/LOCQ)]

(b) A system comprising a monatomic ideal gas initially occupies $1\,m^3$ when the pressure is $400\,kPa$. The system now goes through a reversible isobaric process with the heat input being 100 kJ. Determine the final volume and the work done on the gas. [(CO2)(Evaluating/HOCQ)]

(c) Prove that: (i) $TdS=C_VdT+T\left(\frac{\partial P}{\partial T}\right)_V dV$ (ii) $TdS=C_PdT-T\left(\frac{\partial V}{\partial T}\right)_P dP$.

Where the symbols have their usual meanings. [(CO2)(Evaluating/HOCQ)]

(2 + 2) + (2 + 2) + (2 + 2) = 12

3\. (a) Consider a system consisting of a van der Waals gas. Find the work done by the gas as it expands reversibly from a volume V_1 to a volume V_2: (a) at constant pressure P, (b) at constant temperature T. [(CO2)(Remember/LOCQ)]

(b) The Gibbs function of one mole of a certain gas is given by $g=RT\,lnP+A+BP+CP^2/2+DP^3/3\text{m}$ where A,B,C,D are constants. Find the equation of state of the gas. [(CO2)(Remember/LOCQ)]

(c) Using the Maxwell relation $\left(\frac{\partial S}{\partial V}\right)_T=\left(\frac{\partial P}{\partial T}\right)_V$, find an expression for $\left(\frac{\partial C_V}{\partial V}\right)_T$. Now show that for a van der Waals gas, C_V is a function of T only.

[(CO2)(Remember/LOCQ)]

(d) What is Clausius-Clapeyron equation for the slope of the phase boundary? Find the equation of the vaporization curve by integrating this equation.

[(CO2)(Remember/LOCQ)]

(2 + 2) + 2 + (1.5 + 1.5) + (1 + 2) = 12

Group - C

4\. The eigensates of a quantum system corresponding to the eigenvalues $\frac{\hbar\omega}{2}$ and $\frac{3\hbar\omega}{2}$ are given by $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$.

(a) (i) Construct the Hamiltonian H_0 of the system.

(ii) Construct an operator A that has two eigenvalues μ and ρ and commutes with the Hamiltonian H_0.

(iii) Define linear momentum operator in quantum mechanics.
[(CO1)(Apply/IOCQ)]

(b) The quantum state of a system is given by $|\psi\rangle = \frac{1}{\sqrt{3}}|\varphi_1\rangle + \sqrt{\frac{2}{3}}|\phi_2\rangle$ where $|\varphi_1\rangle$ and $|\varphi_2\rangle$ are the eigenstates of the system.
Determine the probabilities of finding the system in the respective eigenstates.
[(CO1)(Evaluate/HOCQ)]

(c) Show that the eigenvalues of a hermitian operator are real.
[(CO1)(Understand/LOCQ)]

(3 + 2 + 2) + 2 + 3 = 12

5. (a) The eigen function corresponding to n-th excited state for a particle in a one dimensional box $0 \leq x \leq a$ is $\psi_n(x) = \sqrt{\frac{2}{a}} sin\frac{n\pi x}{a}$.
(i) Find the energy difference between the n-th and (n+1)-th eigen states.
(ii) Find the probability density of the n-th state. [(CO1)(Remember/LOCQ)]

(b) Given the operators $\sigma_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\sigma_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. Find the uncertainty product $\Delta\sigma_1\Delta\sigma_2$ for the state $\frac{1}{2}\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$. [(CO1)(Remember/LOCQ)]

(c) Prove the Leibniz identity $[A, BC] = [A, B]C + B[A, C]$ where $[X, Y] = XY - YX$.
[(CO1)(Evaluate/HOCQ)]

(3 + 2) + 4 + 3 = 12

Group - D

6. (a) Show (120) and $(1\bar{1}0)$ planes in Cartesian system by drawing a unit cell in each case. [(CO4)(Remember/LOCQ)]

(b) Find out *h*, *k*, *l* values for (hkl) planes with an interplanar spacing of 1.931 Å in cubic α-Po, having *a* = 3.345 Å. [(CO4)(Remember/LOCQ)]

(c) What is the lattice type and the atom positions in a basis (motif) of a diamond crystal? [(CO4)(Remember/LOCQ)]

(d) Determine linear density of atoms of bcc Ba along [111] and [110] direction. Atomic volume of Ba is 39.24 cm^3/mole. [(CO4)(Evaluate/HOCQ)]

(2 + 2) + 2 + 2 + (2 + 2) = 12

7. (a) Why the diffraction pattern does not contain lines such as (100), (300), (111) or (210) for a bcc lattice? [(CO4)(Remember/LOCQ)]

(b) Show that the reciprocal lattice vector $\vec{G} = h\vec{b_1} + k\vec{b_2} + l\vec{b_3}$ is perpendicular to (*hkl*) plane, where $\vec{b_1}$, $\vec{b_2}$ and $\vec{b_3}$ are three reciprocal primitive vectors. Show that the distance between two adjacent parallel planes of the lattice is $2\pi/|\vec{G}|$.
[(CO4)(Understand/LOCQ)]

(c) A beam of X-rays of wavelength 0.154 nm is diffracted by (110) plane of fcc rock salt with lattice constant of 0.28 nm. Estimate the glancing angle for the second-order diffraction. [(CO4)(Evaluate/HOCQ)]

(d) Develop the equation $2d\ \sin\theta = n\lambda$ from Bragg condition written in terms of wave vector (X-ray) and reciprocal lattice vector. [(CO4)(Apply/IOCQ)]

3 + (3 + 2) + 2 + 2 = 12

Group - E

8. (a) The solution of Schrodinger equation for a one-dimensional periodic lattice is given by $P\frac{\sin\alpha a}{\alpha a} + \cos\alpha a = \cos\alpha a$ with $\alpha^2 = \left(\frac{2mE}{\hbar^2}\right)$.

The symbols have their usual significance. Discuss the formation of Energy bands in a solid. [(CO5)(Create/HOCQ)]

(b) Prove that the effective mass of an electron in any energy band is given by $\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k^2}$. [(CO5)(Create/HOCQ)]

(c) Utilize the Fermi Dirac distribution function to develop an expression the density of holes in the valence band of an intrinsic semiconductor in a small interval of energy. Hence derive the effective concentration of holes in the valence band edge. [(CO5)(Apply/IOCQ)]

3 + 3 + 6 = 12

9. (a) What is phonon ? What are the different types of phonon? Why is it important in studying lattice vibration? [(CO3)(Remember/LOCQ)]

(b) Starting from the basic assumption of lattice vibration develop the dispersion relation between the angular frequency ω and the wave vector k for a two dimensional periodic lattice. [(CO3)(Create/HOCQ)]

(c) At long wavelength limit distinguish between acoustic branch and optical branch. [(CO3)(Analyse/IOCQ)]

(1 + 1 + 1) + 5 + 4 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	45.83	22.92	31.25

Course Outcome (CO):

After the completion of the course students will be able to

1. To understand the concept of basic quantum methods and quantum mechanics of a particle in some specific situations.
2. To understand how thermodynamics gives rise to completely general relationships among various material properties regardless of microscopic structure.
3. To understand the physics behind phonons and lattice vibration and the origin of specific heat.
4. To understand the physics behind X-ray diffraction in crystalline structure of a material, and the different imperfection in it.
5. To understand the basic difference between the atomic structure of an isolated atom and atoms in solids differ and accordingly assures the electrical and thermal properties of solids. To study the energy band formation in solids and the behavior of electron and hole in the bands.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question