COMPUTATIONAL MATHEMATICS
(MATH 3221)

Time Allotted : 3 hrs. **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The Eulerian number $E(5, 4)$ is
(a) 5 (b) 4 (c) 1 (d) 9.

(ii) The Stirling number of the second kind (or Stirling subset number) $S_2(3, 2)$ is
(a) 1 (b) 2 (c) 6 (d) 3.

(iii) The second Bernoulli number B_2 is
(a) $\frac{1}{6}$ (b) $-\frac{1}{2}$ (c) 0 (d) $-\frac{1}{30}$

(iv) The greatest common divisor of the Fibonacci numbers F_{20} and F_{21} is
(a) 3 (b) 2 (c) 1 (d) 5

(v) The remainder in the division of $1!+2!+3!+4!+5!+\ldots+120!$ by 10 is
(a) 3 (b) 4 (c) 1 (d) 2

(vi) $C(6, 0)-C(6, 1)+C(6, 2)-C(6, 3)+C(6, 4)-C(6, 5)+C(6, 6)=$
(a) 64 (b) 0 (c) 6 (d) 1.

(vii) Which one of the following is a quadratic residue mod 11?
(a) 2 (b) 5 (c) 6 (d) 7

(viii) Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x. If $c > 0$ and $\lfloor \sqrt[40]{c} \rfloor = 0$, then
(a) $3 < c < 4$ (b) $c < 1$ (c) $1 < c < 2$ (d) $c > 4$

(ix) The generating function of the sequence $na^n =$
(a) $\frac{ax}{(1-ax)^2}$ (b) $\frac{ax}{(1-ax)^3}$ (c) $\frac{ax}{(1-ax)}$ (d) $\frac{1}{(1-ax)^3}$

(x) The coefficient of x^{10} in $(1 + x + x^2 + \cdots)^2$ is
(a) 11 (b) 12 (c) 10 (d) 9.

Group – B

2. (a) Let T_n denote the minimum number of moves that will transfer n disks from one peg to another in the puzzle called the Tower of Hanoi. Prove that $T_n \le 2T_{n-1}+1$, for $n>0$. Explain why equality may not hold.
[(MATH3221.1, MATH3221.5, MATH3221.6)(Understand/LOCQ)]

(b) Find the sum of the arithmetic progression $\sum_{0\le j\le n}(a+b_j)$ by showing that it is equal to $(n+1)(2a+bn)-\sum_{0\le j\le n}(a+b_j)$.
[(MATH3221.1, MATH3221.5, MATH3221.6)(Analyse/IOCQ)]

7 + 5 = 12

3 (a) Solve the recurrence relation $Q_0=\alpha; Q_1=\beta; Q_n=\dfrac{1+Q_{n-1}}{Q_{n-2}}$, *for* $n>1$. Assume that $Q_n \ne 0$ *for all* $n\ge 0$. [(MATH3221.1, MATH3221.5, MATH3221.6)(Analyze/IOCQ)]

(b) Calculate $\Delta^4(x^{\underline{m}})$ and $\nabla^2(x^{\overline{m}})$.
[(MATH 3221.1, MATH 3221.5, MATH 3221.6)(Understand/LOCQ)]

6 + 6 = 12

Group – C

4. (a) Prove the following recurrence relation for $S_1(n,k)$, the Stirling numbers of the first kind.
$S_1(n,k)=(n-1)S_1(n-1,k)+S_1(n-1,k-1)$, integer $n>0$.
[(MATH 3221.2, MATH 3221.5, MATH 3221.6)(Apply/IOCQ)]

(b) (i) State the definition of the Euler numbers $E(n,k)$ and the recurrence relation satisfying them. Calculate $E(4,0), E(4,1), E(4,2), E(4,3), E(4,4)$.

(ii) Prove that $E(n,k)=E(n,n-1-k)$, integer $n>0$. Use this result to calculate $E(15,14)$. [(MATH3221.2, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

6 + 6 = 12

5. (a) Let F_n denote the Fibonacci numbers. Prove that
(i) $F_{n+5}=5F_{n+1}+3F_n$, (ii) $F_{n-4}=-3F_{n+1}+5F_n$.
[(MATH3221.2, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

(b) Prove that
$z\cot z = \sum_{n\ge 0}(-4)^n B_{2n}\dfrac{z^{2n}}{(2n)!}$
[(MATH 3221.2, MATH 3221.5, MATH 3221.6)(Evaluate/HOCQ)]

(3 + 3) + 6 = 12

Group – D

6. (a) Let n be any positive integer and x be any real number. Prove that a necessary and sufficient condition for the result $\lfloor nx\rfloor = n\lfloor x\rfloor$ is $\{x\}<\dfrac{1}{n}$, where $\{x\}=x-\lfloor x\rfloor$ is the fractional part of x.
[(MATH3221.3,MATH3221.5,MATH3221.6)(Evaluate/HOCQ)]

(b) Find the remainder in the division of $4^{901} + 3^{602} + 29!$ by 31. State every result that you use.

[(MATH 3221.3, MATH 3221.5, MATH 3221.6) (Remember/LOCQ)]

6 + 6 = 12

7. (a) Use the Euclidean algorithm to find a solution, in integers, of the equation $34x + 21y = 3$. [(MATH3221.3, MATH3221.5, MATH3221.6)(Apply/IOCQ)]

(b) Prove that $n^{n/2} \leq n! \leq \frac{(n+1)^n}{2^n}$, where n is a positive integer.

[(MATH 3221.3, MATH 3221.5, MATH 3221.6)(Evaluate/HOCQ)]

5 + 7 = 12

Group – E

8. (a) In how many ways can a person climb up a flight of n steps if the person can skip at most one step at a time? If we assume that the person may take either 1, 2, *or* 3 steps in each stride, find a recurrence relation for the number of ways the person can climb n steps.

[(MATH3221.4, MATH3221.5, MATH3221.6)(Evaluate/HOCQ)]

(b) Prove that the generating function for the Fibonacci numbers is $\frac{z}{1-z-z^2}$.

[(MATH 3221.3, MATH 3221.5, MATH 3221.6) (Apply/IOCQ)]

6 + 6 = 12

9. (a) Solve the recurrence relation $a_n - 7a_{n-1} + 10a_{n-2} = 0$ *for* $n \geq 2$.

[(MATH3221.4, MATH3221.5, MATH3221.6) (Create/HOCQ)]

(b) Find the form of a particular solution to $a_n - 5a_{n-1} + 6a_{n-2} = n^2 4^n$ for $n \geq 2$.

[(MATH 3221.4, MATH 3221.5, MATH 3221.6) (Understand/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	26.04	35.42	38.54

Course Outcome (CO):

After the completion of the course students will be able to

MATH3221.1. Identify patterns in data in the form of recurrences and using the latter to evaluate finite and infinite sums.

MATH3221.2. Explain combinatorial phenomena by using binomial coefficients, generating functions and special numbers.

MATH3221.3. Solve computational problems by applying number theoretic concepts such as primality, congruences, residues etc.

MATH3221.4. Analyze the properties of networks by invoking graph theoretic concepts such as connectivity, matchings, colouring etc.

MATH3221.5. Combine the concepts of recurrences, sums, combinatorics, arithmetic and graph theory in order to comprehend computational methods.

MATH3221.6. Interpret mathematically the algorithmic features of computational situations.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question