ADVANCED NUMERICAL METHODS (MATH 2202)

Time Allotted : 3 hrs. **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In Gauss-Jordan method, the given system of linear equations represented by $AX = B$ is converted to another system $CX = Y$ where C is
(a) diagonal matrix (b) identity matrix
(c) upper triangular matrix (d) lower triangular matrix.

(ii) Taking $h = 1$, the value of $\Delta^n e^x$ is
(a) $e^n e^x$ (b) $(e+1)^n e^x$
(c) $(e-1)^n e^x$ (d) $(e^n - 1)e^x$.

(iii) A system of linear equation can be solved by Cholesky factorization if the coefficient matrix of the system of linear equation is
(a) symmetric and negative definite (b) symmetric and positive definite
(c) diagonal and negative definite (d) diagonal and positive definite.

(iv) In an iterative method, the amount of computation depends on the
(a) number of variables (b) round off errors
(c) ease of using the operators (d) degree of accuracy.

(v) The interval which contains all the eigenvalues of the symmetric matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is
(a) $[0, 9]$ (b) $[0, 6]$ (c) $[1, 9]$ (d) $[1, 6]$.

(vi) The eigenvalue of the matrix $C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ corresponding to the eigen vector $X = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ is given by
(a) 0 (b) 1 (c) 2 (d) 3.

(vii) If $y = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$, then $\Delta^n y = ?$
(a) $a_0 (n-1)!\, h^n$ (b) $a_1 n!\, h^{n-1}$
(c) $a_0 n!\, h^n$ (d) $a_1 n!\, h^n$.

(viii) Which one is not correct?
(a) $E = 1 + \Delta$ (b) $E^{-1} = 1 + \nabla$
(c) $E\ y_x = y_{x+h}$ (d) $E^{-1} y_x = y_{x-h}$

(ix) $[x_0, x_1, x_2] = ?$
(a) $\frac{[x_1,x_2]-[x_0,x_1]}{x_0-x_2}$ (b) $\frac{[x_0,x_2]-[x_0,x_1]}{x_1-x_2}$
(c) $\frac{[x_0,x_1]-[x_1,x_2]}{x_1-x_2}$ (d) $\frac{[x_1,x_2]-[x_0,x_1]}{x_2-x_0}$

(x) Out of the following algorithms, which one has the slowest rate of convergence?
(a) Dichotomous search (b) Fibonacci search
(c) Golden search (d) Interval halving.

Group – B

2. (a) Use Gauss-Seidel method to solve the given system of equations correct up to three decimal points:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25.$$

Take the initial approximation as: $(x, y, z) = (0, 0, 0)$.

[(MATH2202.1, MATH2202.4, MATH2202.6)(Evaluate/HOCQ)]

(b) What do you mean by $||\cdot||_1$ norm of a matrix? Using that norm, find the condition number of the following system of linear equations.

$$2x - y + z = 2$$
$$x + z = 2$$
$$3x - y + 4z = 6$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

6 + 6 = 12

3. (a) Use Gauss-Elimination method to solve the system of equations:

$$2x_1 + x_2 - 3x_3 - x_4 = -1$$
$$4x_1 + 2x_2 - x_3 + 2x_4 = 7$$
$$6x_1 - 4x_2 - 2x_3 + 4x_4 = 4$$
$$x_1 + x_2 + x_3 - x_4 = 2.$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Apply/IOCQ)]

(b) Can we solve the following system of linear equations using Cholesky factorization method? Justify your answer.

$$2x - y = 1$$
$$-x + 2y - z = 0$$
$$-y + 2z = 1$$

[(MATH2202.1, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

6 + 6 = 12

Group – C

4. (a) Suppose the eigenvalues of a 4×4 symmetric matrix A are to be computed. Because of a data entry error, every entry of A has 0.0001 added to it, leading to

another matrix $\hat{A}$? What is the error bound for $|\lambda_k - \widehat{\lambda_k}|$ as given by the Stability corollary? How does the error bound change if the matrix was of order 5×5? Justify.

[(MATH2202.3, MATH2202.4, MATH2202.6)(Understand/LOCQ)]

(b) Compute five iterations of the power method to approximate a dominant eigenvector of $A = \begin{bmatrix} -2 & -3 \\ 6 & 7 \end{bmatrix}$ starting with initial approximation $X_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Using Rayleigh quotient, approximate the dominant eigenvalue of A.

[(MATH2202.3, MATH2202.4, MATH2202.6) (Understand/LOCQ)]

4 + 8 = 12

5. Find the Singular Value Decomposition (SVD) of the matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

[(MATH2202.3, MATH2202.4, MATH2202.6) (Apply/IOCQ)]

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Group – D

6. (a) Compute $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by (i) Simpson's 1/3rd rule,
(ii) Weddle's rule,
taking $h = 0.26179 = 15°$. [(MATH2202.2, MATH2202.6)(Apply/IOCQ)]

(b) Find $f(x)$ as a polynomial in x by using the Newton's divided difference interpolation formula for the following table:

x	-1	0	1	2	3	4
$f(x)$	-16	-7	-4	-1	8	29

[(MATH2202.2, MATH2202.6)(Remember/LOCQ)]

(3 + 3) + 6 = 12

7. (a) Given the values

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

Evaluate $f(9)$ using Newton's divided difference formula.

[(MATH2202.2, MATH2202.6) (Evaluate/HOCQ)]

(b) Obtain the cubic spline corresponding to the interval [2,3] for the following table:

x	0	1	2	3
$f(x)$	1	2	33	244

with $M_0 = 0, M_3 = 0$.
Hence evaluate $f(2.5)$.

[(MATH2202.2, MATH2202.6) (Apply/IOCQ)]

5 + 7 = 12

Group – E

8. Execute Fibonacci search algorithm to minimize $f(x) = x^4 - 13x^3 + 62x^2 - 79x$ over [0,2] taking 6 functional evaluations. [(MATH2202.5, MATH2202.6) (Apply/IOCQ)]

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9. Use Dichotomous Search algorithm to minimize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ over $[0, 2]$, tolerance limit being 0.3. Consider $\epsilon = 0.001$.
[(MATH2202.5, MATH2202.6) (Apply/IOCQ)]
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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	31.25%	57.29%	11.46%

Course Outcome (CO):

After the completion of the course students will be able to
MATH2202.1 Analyze certain algorithms, numerical techniques and iterative methods that are used for solving system of linear equations.
MATH2202.2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation, integration and differentiation.
MATH2202.3 Apply the knowledge of matrices for calculating eigenvalues and eigenvectors and their stability for reducing problems involving Science and Engineering
MATH2202.4 Develop an understanding to reduce a matrix to its constituent parts in order to make certain subsequent calculations simpler.
MATH2202.5 Apply various optimization methods for solving realistic engineering problems.
MATH2202.6 Compare the accuracy and efficiency of the above-mentioned methods.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question