PHYSICS I
(PHYS 1001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The coordinate (4, θ, 0) in spherical polar coordinate system is a point in the Cartesian plane
(a) XY (b)YZ
(c) XZ (d) at the point of intersection of XZ and XY.

(ii) A vector field is given by $\vec{F} = \hat{\imath} \times \vec{r}$. where $\hat{\imath}$ is the unit vector along X axis and $\vec{r}$ is the position vector in Cartesian coordinate system. The field $\vec{F}$ is a
(a) source field (b) sink field
(c) solenoid field (d) conservative field.

(iii) The value of k for the given irrotational field $\vec{A} = 2\hat{\imath} + (2k-1)x\hat{\jmath}$ is
(a) 2 (b) 1/2
(c) 0 (d) 1.

(iv) The unit vector along the increasing direction of r in plane polar coordinates (r, θ) at an angle $\theta = \frac{2\pi}{3}$ is
(a) $\frac{1}{2}(\hat{\imath} + \hat{\jmath})$ (b) $\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath})$
(c) $\frac{1}{\sqrt{3}}(-\hat{\imath} + \hat{\jmath})$ (d) $\frac{1}{2}(-\hat{\imath} + \sqrt{3}\,\hat{\jmath})$

(v) The volume element of spherical polar coordinate is
(a) $drd\theta d\varphi$ (b) r sin$\theta drd\theta d\varphi$
(c) r^2sin$\theta drd\theta d\varphi$ (d) $rdrd\theta d\varphi$

(vi) A damped oscillator of frequency ω is acted upon by an external force $F = F_o sin\omega t$. If ω_o is the natural frequency of oscillation, the frequency of oscillation of the system after long duration is
(a) ω_o (b) $\omega - \omega_o$
(c) ω (d) $\frac{\omega + \omega_o}{2}$

(vii) During lasing action
(a) only stimulated emission takes place
(b) both spontaneous and stimulated emission takes place
(c) only spontaneous emission takes place
(d) spontaneous emission is needed for the process.

(viii) A half wave plate introduces a phase difference
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{5}$ (c) π (d) 2π

(ix) A dielectric material is placed in vacuum in a uniform electric field of $\vec{E} = 4$ V/m. what is the electric field inside the material if the relative permittivity of the dielectric material is 2.
(a) 2V/m (b) 4V/m (c) 8V/m (d) 0

(x) The relative permeability of a medium is equal to (with M = magnetization of the medium and H = magnetic field strength)
(a) $1 + \frac{M}{H}$ (b) $1 - \frac{M}{H}$ (c) $1 + \sqrt{\frac{M}{H}}$ (d) $1 - \sqrt{\frac{M}{H}}$

Group- B

2. (a) The vector function $\vec{A} = 2x\hat{\imath} + 3y\hat{\jmath} + 4z\hat{k}$ and the scalar function $u(x, y, z) = x^2 + y^2 + z^2$, then find the value of $div(u\vec{A})$ at (1,1,1). [(CO1)(Remember/LOCQ)]

(b) For the set of unit vectors $(\hat{r}, \widehat{\theta}, \widehat{\varphi})$ in spherical polar coordinate system, show that $\dot{\hat{r}} = \dot{\theta}\widehat{\theta} + \sin\theta\, \dot{\varphi}\widehat{\varphi}$. [(CO1)(Apply/IOCQ)]

(c) Show that if $\vec{\nabla} \times \vec{F} = 0$ and $\vec{\nabla} \times \vec{B} = 0$, then $\vec{F} \times \vec{B}$ is solenoidal , where $\vec{F}$, $\vec{B}$ are given vector functions . [(CO1)(Apply/IOCQ)]

(d) A scalar function $\rho(x, y) = In(\sqrt{x^2 + y^2})$, calculate $\vec{\nabla}\rho$ and express your answer in (r,θ) system . Draw the vector field. [(CO1)(Apply/IOCQ)]

(e) If $\vec{A}$ is the vector from a point (x_1, y_1, z_1) to another (x_2, y_2, z_2), evaluate $\nabla.(\nabla \times \vec{A})$. [(CO1)(Apply/IOCQ)]

2 + 2 + 2 + (2 + 1 + 1) + 2 = 12

3. (a) Show that for a particle moving in a central force field, the areal velocity is constant. [(CO2)(Remember/LOCQ)]

(b) The orbit of a particle under the influence of a central force is given by $r = ae^{b\theta}$, where a, b are constant. Find the corresponding force law. [(CO2)(Understand/IOCQ)]

(c) Derive the expression of coriolis theorem. Show that the rate of change of magnitude of any vector is independent of frame of reference. [(CO2)(Analyse/IOCQ)]

(d) Show for a particle at rest on the surface of earth $\overrightarrow{g_{eff}} = \vec{g} - \vec{\omega} \times (\vec{\omega} \times \vec{r})$, hence discuss the oblate shape of the surface of earth. [(CO2)(Apply/IOCQ)]

2 + 3 + (3 + 1) + 3 = 12

Group - C

4. (a) The damped frequency of vibration of a body is 200Hz. The amplitude of vibration becomes $\frac{1}{e}$ of the initial amplitude after 1 second. Calculate the frequency of free vibration. [(CO4)(Remember/LOCQ)]

(b) Show following mechanical oscillation that for series LCR circuit the maximum potential across the capacitor happens when $\omega = \omega_0 \left(1 - \frac{1}{2Q^2}\right)^{\frac{1}{2}}$ and across the inductor occur when $\omega = \omega_0 \left(1 - \frac{1}{2Q^2}\right)^{-\frac{1}{2}}$. [(CO3)(Apply/IOCQ)]

(c) A particle of 2gm mass is subjected to an elastic force of 0.03 Nm^{-1} and frictional force of 5×10^{-4} $N/_{cms^{-1}}$. If the particle is displaced through 2 cm and then released decide whether the resulting motion is oscillatory or not. If so, find its time period. [(CO3)(Evaluate/HOCQ)

(d) Show that the ratio between average input power and the average dissipated power is unity. [(CO3)(Apply/IOCQ)]

2 + 3 + (2 + 1) + 4 = 12

5. (a) (i) A plane progressive radio wave follows the expression $\psi(\vec{r}, t) = \psi_0 \sin(\vec{k}.\vec{r} - \omega t)$with amplitude $\psi_0 = 6$m.It is propagating along the direction of the vector $3\hat{\imath} + \sqrt{2}\hat{\jmath} + \sqrt{5}\hat{k}$. Consider $\lambda = 40$m and $\omega = 43\pi/$ 80Hz as wave length and frequency of the wave respectively. Construct the wave function representing the wave. Find the value of the wave function at a point (1,0,0) at $t = 1$ s. [(CO6)(Understand/LOCQ)]

(ii) Explain the fact that $\psi(x, t) \doteq \psi(x + ct)$ indeed represents a wave propagating along the negative x direction. (Symbols have their usual meaning) [(CO6)(Analyze/IOCQ)]

(b) (i) A plane polarized light vector is given by $\vec{E}(\vec{r}, t) = E_0 cos(\vec{k}.\vec{r} - \omega t)(\hat{\imath} + \hat{\jmath} + \hat{k})$ is propagating along the direction $\hat{\imath} - \hat{\jmath}$. Find the equation of the plane of vibration. [(CO6)(Understand/IOCQ)]

(ii) The light vector of a polarized light is given by $\vec{E}(z, t) \doteq 2cos\frac{2\pi}{3}\left(z - 3t - 38i + 2cos2\pi 3z - 3t + 38j\right.$. Identify the state of polarization. [(CO6)(Understand/LOCQ)]

(c) Explain population inversion. [(CO6)(Remember/LOCQ)]

(3 + 1 + 2) + (2 + 2) + 2 = 12

Group - D

6. (a) Suppose there are two small point objects located on the Y-axis. One object has a charge of +q and is located at the coordinates (0, +a). The other point object has a charge of –q and is located at the coordinates (0, -a). Find the electric field vector for locations on the X-axis (x, 0) such that |x| > a. [(CO4)(Remember/LOCQ)]

(b) Consider the charge distribution which consists of two uniformly charged thin rods, each of length l, attached end to end. The one on the left has a total charge of –Q and the one on the right has a charge of +Q. They lie on the X-axis with their common end at x=0. Find an expression for the electric field at locations (0, y) on the Y-axis. [(CO4)(Understand/HOCQ)]

(c) What is the electric potential difference between the origin (0,0) and the position (2,3) meters if the electric field in this region is equal to $\vec{E}(\vec{r}) = E_0\hat{\imath}$, where E_0 has units of volts/meter? [(CO4)(understand/IOCQ)]

(d) Electric potential is given (in spherical coordinates) by $V = \frac{a}{r}cos\theta\ sin\varphi$, where a is constant. Calculate the electric field at point $(1,\frac{\pi}{3},\frac{\pi}{6})$. [(CO4)(understand/IOCQ)]

3 + 4 + 2 + 3 = 12

7. (a) Two parallel metal plates are placed along xy plane at equal distances from z = 0 plane on its either side. The upper plate is kept at potential +20 volt and the lower one is grounded. If the upper plate's potential is increased by 10 volt, keeping the lower one still grounded and fixed, calculate what should be the position of the upper plate in terms of the initial position if the electric field has to be kept unchanged. [(CO1)(Remember/LOCQ)]

(b) For a linear isotropic dielectric medium express the electric displacement vector $\vec{D}$ both in terms of polarization vector and electric field vector**.** [(CO5)(Understand/LOCQ)]

(c) A ring of radius R has linear charge density $\lambda = \lambda_0 cos^2\varphi$. Find the electric potential at a point (0,0,z) on the axis of the ring. Consider origin at the centre of the ring. [(CO4)(Remember/LOCQ)]

(d) A dielectric sphere of radius r, centered at the origin, carries a polarization $\vec{P} = k\vec{r}$. Calculate the total volume bound charge and the total surface bound charge. [(CO1)(Analyze/IOCQ)]

3 + 3 + 3 + 3 = 12

Group – E

8. (a) Show that the vector field$\vec{B} = a(x^2\hat{\imath} + by\hat{\jmath})$, (where, a and b are constant) does not represent a magnetostatic field. [(CO4)(Remember/LOCQ)]

(b) A current carrying circular loop (with the centre at the origin) on XY-plane has the magnetic moment $\vec{m} = m\hat{k}$. The vector potential $\overrightarrow{A}$ due to it at a distance $\vec{r}$is given by $\frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$. Find the magnetic field at a point P with position vector $\vec{r} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$. [(CO4)(Analyse/IOCQ)]

(c) Given two vector potentials $\overrightarrow{A_1} = a(-y\hat{\imath} + z\hat{\jmath})$ and $\overrightarrow{A_2} = a[(x-y)\hat{\imath} + (y+z)\hat{\jmath} + z\hat{k}]$ represent the same magnetic field. Explain the reason. [(CO4)(Understand/IOCQ)]

(d) From the concept of charge conservation show that $\vec{\nabla}.\vec{j} + \frac{\partial\rho}{\partial t} = 0$, where the symbols have their usual meaning. [(CO4)(Apply/IOCQ)]

3 + 3 + 3 + 3 = 12

9. (a) Explain the meaning of free and bound current with reference to magnetised magnetic material. Show that curl of magnetic field intensity depends only on the free current. [(CO6)(Remember/LOCQ), (Apply/IOCQ)]

(b) A dipole magnetic of moment $m(\hat{\imath} + \hat{\jmath})$ is placed in a magnetic field $\vec{B} = -\vec{B}_0\hat{\imath}$. Calculate the torque generated. [(CO4)(Apply/IOCQ)]

(c) Starting from the expression for vector potential and considering cylindrical symmetry evaluate for the expression for the magnetic field due to a current carrying infinite conductor. [(CO4)(Evaluate/HOCQ)]

(d) Show that the vector $Cz\hat{\imath}$ represents a magnetostatic field where, C is a constant. Show that the corresponding current density represents a steady current. [(CO5)(Apply/IOCQ)]

(2 + 2) + 2 + 4 + 2 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	34.37	54.17	11.46

Course Outcome (CO):

After the completion of the course students will be able to

1. Understand and apply Vector Calculus as tool for solving different physical problems.
2. Analyze the nature of central forces and rotating frame phenomenon to understand basic space science and real world applications.
3. Interpret the different types of oscillatory motion and resonance
4. Apply fundamental theories and technical aspect in the field of electricity and magnetism in solving real world problems in that domain.
5. Understand the Electrical and Magnetic properties of different types of materials for scientific and technological use.
6. Develop Analytical & Logical skill in handling problems in technology related domain.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question