

MATHEMATICS - II
(MATH 1201)

Time Allotted : 3 hrs.

Full Marks : 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group - A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) A random variable X has the following probability density function:

$$f(x) = \begin{cases} 1, & \text{for } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$
 The mean of X is
 (a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 1
- (ii) The variance of first n positive integers is
 (a) $\frac{n+1}{2}$ (b) $\frac{n^2+1}{2}$ (c) $\frac{n^2-1}{12}$ (d) 0
- (iii) Newton-Raphson's method is also known as
 (a) normal method (b) tangent method
 (c) parallel method (d) chord method.
- (iv) Simpson's $\frac{1}{3}$ rule gives exact result for polynomial up to order
 (a) 2 (b) 3 (c) 4 (d) 5
- (v) Modified Euler method of order four is analogous with Taylor series expansion up to first
 (a) 2 terms (b) 3 terms (c) 4 terms (d) 5 terms.
- (vi) BFS and DFS algorithms are used to find
 (a) a spanning tree of a weighted graph
 (b) a spanning tree of an unweighted graph
 (c) shortest distance between any two vertices
 (d) isomorphisms of graph.
- (vii) The number of pendant vertices in a binary tree having n vertices
 (a) $\frac{n+1}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n(n+1)}{2}$ (d) n

- (viii) A tree having no cut vertex is a graph of
 (a) three vertices (b) two vertices
 (c) two edges (d) three edges.
- (ix) Let $I = \int_0^1 f(x)dx$, where $f(x) = x^n \log x$ ($n > 0$), then
 (a) I is proper (b) I is improper
 (c) $f(x)$ has infinite discontinuity at $x = 0$ (d) both (b) and (c).
- (x) $L^{-1}\left(\frac{24}{(p+1)^5}\right) =$
 (a) $\frac{24t^3}{e^t}$ (b) $\frac{24t^4}{e^t}$ (c) $\frac{t^4}{e^{-t}}$ (d) $\frac{t^4}{e^t}$

Group - B

2. (a) Only 1 in 1000 adult is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that, when an individual actually has the disease, a positive result will diagnostic 99% of the time, while an individual without the disease will show a positive test result only 2% of the time. What is the probability of an individual having positive result? What is the probability an individual with positive result has the disease?
 [(MATH1201.1, MATH1201.2) (Understand/LOCQ)]
- (b) Let's say that 80% of all business startups in the IT industry report that they generate a profit in their first year. If a sample of 10 new IT business startups is selected, find the probability that exactly seven will generate a profit in their first year.
 [(MATH1201.1, MATH1201.2) (Apply/IOCQ)]
6 + 6 = 12

3. (a) Determine the mean and variance of the random variable X having the following probability distribution.

$X = x$	1	2	3	4	5	6	7	8	9	10
$P(x)$	0.15	0.10	0.10	0.01	0.08	0.01	0.05	0.02	0.28	0.20

- [(MATH1201.1, MATH1201.2) (Understand/LOCQ)]
- (b) The average number of acres burned by forest and range fires in a large New Mexico County is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given year? (Given $P(Z < -2.40) = 0.0082$) and $P(Z < -0.13) = 0.4483$)
 [(MATH1201.1, MATH1201.2) (Analyze/IOCQ)]
6 + 6 = 12

Group - C

4. (a) Solve the following system of equations using Gauss-Seidel method. (Assume $x_0^{(0)} = y_0^{(0)} = z_0^{(0)} = 0$).
- $$6x + 15y + 2z = 72$$
- $$x + y + 54z = 110$$
- $$27x + 6y - z = 85$$

[(MATH1201.3) (Apply/IOCQ)]

- (b) Given that $\frac{dy}{dx} = \log_{10}(x + y)$, $y(0) = 1$, find $y(0.2)$ using modified Euler's method.

[(MATH1201.3) (Apply/LOCQ)]

6 + 6 = 12

5. (a) Using the Newton-Raphson method to find a root of the equation $x^3 - 3x - 5 = 0$.

[(MATH1201.3) (Remember/LOCQ)]

- (b) Using Simpson's $\frac{1}{3}$ rule, evaluate the following integral. Take $h = 1$.

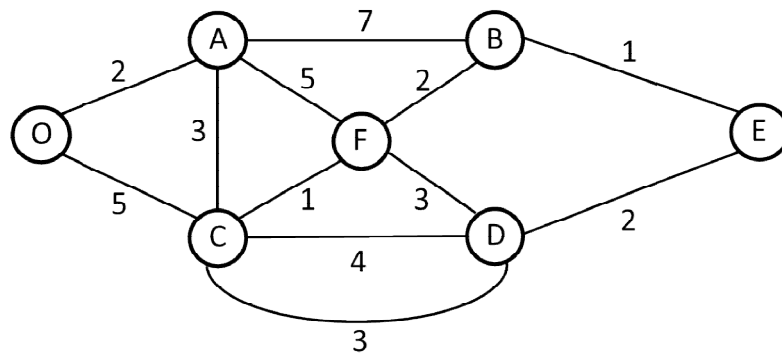
$$\int_0^6 \frac{1}{1+x^2} dx.$$

[(MATH1201.3) (Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Using Dijkstra's algorithm find a shortest path from O to E in the following graph:



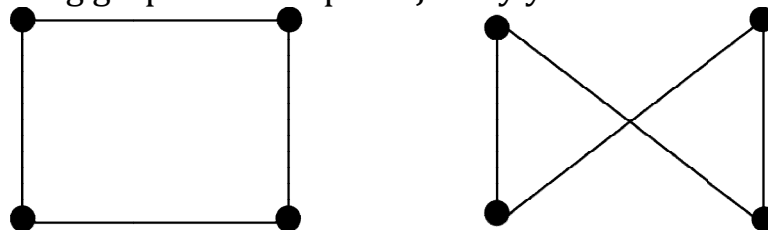
[(MATH1201.4) (Evaluate/HOCQ)]

- (b) Prove that the number of edges in a simple graph with n vertices cannot exceed $n(n-1)/2$.

[(MATH1201.4) (Understand/LOCQ)]

6 + 6 = 12

7. (a) Are the following graphs isomorphic? Justify your answer.



[(MATH1201.3) (Analyze/IOCQ)]

- (b) Draw the graph corresponding to the given incidence matrix:

$$\begin{bmatrix} 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix}$$

[(MATH1201.3) (Create/HOCQ)]

- (c) What is the maximum number of edges in a bipartite graph on 12 vertices?

[(MATH1201.4) (Understand/LOCQ)]

5 + 4 + 3 = 12

Group - E

8. (a) Evaluate the following integral if it exists.

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+2x+2}$$

[(MATH1201.5, MATH1201.6) (Evaluate/HOCQ)]

- (b) Find the Laplace transform of
- $t^2 e^{-2t} \cos t$
- .

[(MATH1201.5, MATH1201.6) (Remember/IOCQ)]

6 + 6 = 12

9. (a) Assuming the convergence of the integral prove that,
- $\int_0^{\infty} \sqrt{x} e^{-x^3} dx = \frac{\sqrt{\pi}}{3}$
- .

[(MATH1201.5, MATH1201.6) (Understand/LOCQ)]

- (b) Solve by Laplace Transform:

$$y''(t) + y(t) = \sin 2t, \text{ where } y(0) = y'(0) = 1.$$

[(MATH1201.5, MATH1201.6) (Apply/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	34.38%	42.7%	22.92%

Course Outcome:

MATH1201. 1. Demonstrate the knowledge of probabilistic approaches to solve wide range of engineering problem.

MATH1201. 2. Recognize probability distribution for discrete and continuous variables to quantify physical and engineering phenomenon.

MATH1201. 3. Develop numerical techniques to obtain approximate solutions to mathematical problems where analytical solutions are not possible to evaluate.

MATH1201. 4. Analyze certain physical problems that can be transformed in terms of graphs and trees and solving problems involving searching, sorting and such other algorithms.

MATH1201. 5. Apply techniques of Laplace Transform and its inverse in various advanced engineering problems.

MATH1201. 6. Interpret differential equations and reduce them to mere algebraic equations using Laplace Transform to solve easily.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question