

**PROBABILITY AND NUMERICAL METHODS  
(MATH 2202)**

**Time Allotted : 3 hrs**

**Full Marks : 70**

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and  
any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group - A  
(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i)  $\Delta \nabla =$  (the notations have their usual meanings)  
(a)  $\Delta - \nabla$                       (b)  $\nabla - \Delta$                       (c)  $\Delta + \nabla$                       (d) 0
- (ii) If a Poisson variate  $X$  is such that  $P(X = 1) = P(X = 2)$ , then  $P(X = 0)$  is  
(a)  $e^{-1}$                       (b)  $e^{-4}$                       (c)  $e^{-2}$                       (d) 1
- (iii) The regression lines between two random variables  $X$  and  $Y$  are given by  $x + 4y + 3 = 0$  and  $4x + 9y + 5 = 0$ . Let  $u = -2x + 5$ ,  $v = -4y + 6$ , then the correlation coefficient between  $u$  and  $v$  is:  
(a) 0.75                      (b)  $-0.75$                       (c)  $-0.5$                       (d) 0.5
- (iv)  $\sin x + e^x = 1$  is  
(a) an algebraic equation                      (b) a linear algebraic equation  
(c) a transcendental equation                      (d) a linear transcendental equation.
- (v) Let  $X$  be a random variable. Then the combination of  $E(X)$  and  $E(X^2)$  which is NOT possible for the random variable  $X$  is  
(a) 0 and 1                      (b)  $\frac{1}{4}$  and  $\frac{1}{5}$                       (c) 4 and 5                      (d)  $\frac{1}{2}$  and  $\frac{1}{3}$
- (vi) Let  $f(x) = 0$  be an equation such that  $f(a) * f(b) > 0$  for two real numbers  $a$  and  $b$ , then which one of the following is true:  
(a) at least one root of  $f(x) = 0$  lies in  $(a, b)$   
(b) no root of  $f(x) = 0$  lies in  $(a, b)$   
(c) an odd number of roots of  $f(x) = 0$  lies in  $(a, b)$   
(d) either no root or an even number of roots of  $f(x) = 0$  lies in  $(a, b)$ .
- (vii) The degree of precision of Simpson's one-third rule is  
(a) 1                      (b) 2                      (c) 3                      (d) 5.



- (b) Two newspapers  $X$  and  $Y$  are published in a certain city. It is estimated from a survey that 16% read  $X$ , 14% read  $Y$  and 5% read both the newspapers. Find the probabilities that a randomly selected person
- (i) does not read any newspaper
  - (ii) reads only  $Y$ .

[(Understand/LOCQ)]

**6 + 6 = 12**

**Group - D**

6. (a) The weight of a student in a college is normally distributed with mean,  $m = 40$ kg, standard deviation,  $\sigma = 5$  kg. Find the percentage of the students that have weight (i) greater than 40 kg (ii) greater than 50 kg (iii) between 38 kg and 52 kg.

Given  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^2 e^{-t^2/2} dt = 0.9772$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^2/2} dt = 0.6554$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2.4} e^{-t^2/2} dt = 0.9918$ .

[(Apply/IOCQ)]

- (b) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible:

Regression equations:  $8x - 10y + 66 = 0$ ,  $40x - 18y - 214 = 0$ .

- (i) Identify which one is the regression line of  $y$  on  $x$ .
- (ii) Find the mean values of  $x$  and  $y$ .
- (iii) Find the correlation coefficient between  $x$  and  $y$ .

[(Apply/IOCQ)]

**6 + 6 = 12**

7. (a) Let a random variable  $X$  has the following probability mass function:

$x$	0	1	2	3	4	5	6	7
$P(X = x)$	0	$k$	$2k$	$2k$	$3k$	$k^2$	$2k^2$	$7k^2 + k$

- (i) Determine the constant  $k$ .
- (ii) Evaluate  $P(3 < X \leq 6)$ .
- (iii) Obtain the distribution function  $F(x)$ .

[(Understand/LOCQ)]

- (b) A radioactive source emits on an average 2.5 particles per second. Use Poisson distribution to calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds.

[(Apply/IOCQ)]

**6 + 6 = 12**

**Group - E**

8. (a) The bivariate probability distribution of the random variables  $X$  and  $Y$  is given below:

$Y \backslash X$		1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	

2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$
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Find

- (i) the marginal probability mass functions of the random variables X and Y,
- (ii)  $P(X \leq 1)$ , and
- (iii)  $P(Y \leq 3)$ . [[Understand/LOCQ]]

- (b) A fair dice is tossed repeatedly. If  $\{X_n\}, n = 1, 2, 3 \dots$  denotes the maximum of the numbers occurring in the first  $n$  tosses, find the transition probability matrix  $P$  of the Markov chain  $\{X_n\}$ . Find also  $P^2$  and  $P(X_2 = 5)$ . [Evaluate/HOCQ]  
**6 + 6 = 12**

9. (a) The two dimensional random variables  $X$  and  $Y$  have a joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

- (i) Find  $P\left(X < \frac{1}{2}, Y < \frac{1}{4}\right)$ .
- (ii) Find all the marginal and conditional probability density functions. [[Evaluate/HOCQ]]

- (b) For the following transition probability matrix for states  $\{0, 1, 2, 3\}$

$$\begin{bmatrix} 0 & 0.2 & 0.8 & 0 \\ 0.3 & 0.1 & 0 & 0.6 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- (i) draw the state transition diagram
- (ii) identify the recurrent, transient and absorbing states, and the communicating classes. [[Evaluate/HOCQ]]  
**6 + 6 = 12**

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	37.5	37.5	25.0

\*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question