# PROBABILITY AND NUMERICAL METHODS (MATH 2202)

**Time Allotted : 3 hrs** 

Full Marks: 70

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

## Group – A (Multiple Choice Type Questions)

Choose the correct alternative for the following:  $10 \times 1 = 10$ 1. (i)  $\Delta$ .  $\nabla$  = (the notations have their usual meanings) (a)  $\Delta - \nabla$ (b)  $\nabla - \Delta$ (d) 0(c)  $\Delta + \nabla$ If a Poisson variate X is such that P(X = 1) = P(X = 2), then P(X = 0) is (ii) (b)  $e^{-4}$ (a)  $e^{-1}$ (c)  $e^{-2}$ (d) 1 The regression lines between two random variables X and Y are given (iii) by x + 4y + 3 = 0 and 4x + 9y + 5 = 0. Let u = -2x + 5, v = -4y + 6, then the correlation coefficient between u and v is: (a) 0.75 (b) -0.75(c) - 0.5(d) 0.5  $\sin x + e^x = 1$  is (iv) (a) an algebraic equation (b) a linear algebraic equation (d) a linear transcendental equation. (c) a transcendental equation Let X be a random variable. Then the combination of E(X) and  $E(X^2)$  which is (v) NOT possible for the random variable X is (b)  $\frac{1}{4}$  and  $\frac{1}{5}$  (c) 4 and 5 (d)  $\frac{1}{2}$  and  $\frac{1}{3}$ (a) 0 and 1 Let f(x) = 0 be an equation such that f(a) \* f(b) > 0 for two real numbers (vi) *a* and *b*, then which one of the following is true: (a) at least one root of f(x) = 0 lies in (a, b)(b) no root of f(x) = 0 lies in (a, b)(c) an odd number of roots of f(x) = 0 lies in (a, b)(d) either no root or an even number of roots of f(x) = 0 lies in (a, b). (vii) The degree of precision of Simpson's one-third rule is (d) 5. (a) 1 (b) 2 (c) 3

(viii)	The probability o	f going from state	1 in period 2 to sta	ate 4 in period 3 is
	(a) p <sub>12</sub>	(b) p <sub>23</sub>	(c) p <sub>14</sub>	(d) p <sub>43</sub>

(ix)	One of the root			
	(a) 0 and 1	(b) 1 and 2	(c) 2 and 3	(d) 3 and 4

(x) Which of the following methods is an iterative method?
 (a) Gauss Elimination method
 (b) Gauss Jordan method
 (c) Gauss-Seidel method
 (d) LU factorization method.

# **Group-B**

2. (a) Solve the following system of equations by Gauss-Seidal iteration method correct to three significant figures:

$$6x + 15y + 2z = 72$$
  

$$x + y + 54z = 110$$
  

$$27x + 6y - z = 85$$

(b) Find a real positive root of the equation  $x^3 - 9x + 1 = 0$  correct upto two decimal places by bisection method. [(Apply/IOCQ)]

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6 + 6 = 12
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3. (a) Find the missing terms in the following table:

x	0	1	2	3	4	5
f(x)	0	$f_1$	8	15	$f_4$	35

[(Evaluate /HOCQ)]

(b) Evaluate  $\int_0^1 \frac{1}{1+x} dx$  by using Simpson's  $\frac{1}{3}$ rd rule taking eleven ordinates and hence find the value of  $\log_e 2$  correct up to five significant figures.

[(Remember/LOCQ)]

6 + 6 = 12

# Group - C

- 4. (a) Two bags contain 5 white, 3 black balls and 10 white, 3 black balls respectively. Two balls are transferred at random from first bag to second bag and then a ball is drawn at random from later. The ball so drawn is found to be white in colour. Find the probability that the transferred balls were both white. [(Apply/IOCQ)]
  - (b) A man alternatively tosses a coin and throws a die beginning with tossing the coin. What is the probability that he will get a head before he gets a 5 or 6 on the die? [(Understand/LOCQ)]

6 + 6 = 12

5. (a) A box contains 5 defective and 10 non-defective lamps. Eight lamps are drawn at random in succession without replacement. What is the probability that the 8th lamp is the 5th defective? [(Understand/LOCQ)]

(b) Two newspapers *X* and *Y* are published in a certain city. It is estimated from a survey that 16% read *X*, 14% read *Y* and 5% read both the newspapers. Find the probabilities that a randomly selected person

(i) does not read any newspaper

(ii) reads only *Y*.

[(Understand/LOCQ)] 6 + 6 = 12

# Group - D

6. (a) The weight of a student in a college is normally distributed with mean, m = 40kg, standard deviation,  $\sigma = 5$  kg. Find the percentage of the students that have weight (i) greater than 40 kg (ii) greater than 50 kg (iii) between 38 kg and 52 kg.

Given  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2} e^{-t^{2}/2} dt = 0.9772$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0.4} e^{-t^{2}/2} dt = 0.6554$ ,  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{2.4} e^{-t^{2}/2} dt = 0.9918.$  [(Apply/IOCQ)]

(b) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible:

Regression equations: 8x - 10y + 66 = 0, 40x - 18y - 214 = 0.

- (i) Identify which one is the regression line of *y* on *x*.
- (ii) Find the mean values of *x* and *y*.

(iii) Find the correlation coefficient between *x* and *y*.

[(Apply/IOCQ)] 6 + 6 = 12

7. (a) Let a random variable *X* has the following probability mass function:

x	0	1	2	3	4	5	6	7
P(X = x)	0	k	2 <i>k</i>	2 <i>k</i>	3 <i>k</i>	$k^2$	$2k^{2}$	$7k^2 + k$

(i) Determine the constant *k*.

(ii) Evaluate  $P(3 < X \le 6)$ .

- (iii) Obtain the distribution function F(x). [(Understand/LOCQ)]
- (b) A radioactive source emits on an average 2.5 particles per second. Use Poisson distribution to calculate the probability that 3 or more particles will be emitted in an interval of 4 seconds. [(Apply/IOCQ)]

6 + 6 = 12

# Group - E

8. (a) The bivariate probability distribution of the random variables *X* and *Y* is given below:

Y	1	2	3	4	5	6
X						
0	0	0	1	2	2	3
			32	32	32	32
1	1	1	1	1	1	1
	16	16	8	8	8	8

2	1	1	1	1	0	2
	32	32	64	64		64

Find

(i) the marginal probability mass functions of the random variables X and Y, (ii)  $P(X \le 1)$ , and

(iii) (iii)  $P(Y \le 3)$ .

[(Understand/LOCQ)]

- (b) A fair dice is tossed repeatedly. If  $\{X_n\}$ , n = 1,2,3... denotes the maximum of the numbers occurring in the first *n* tosses, find the transition probability matrix *P* of the Markov chain  $\{X_n\}$ . Find also  $P^2$  and  $P(X_2 = 5)$ . [Evaluate/HOCQ)] **6** + **6** = **12**
- 9. (a) The two dimensional random variables *X* and *Y* have a joint probability density function

$$f(x, y) = \begin{cases} 8xy, & 0 < x < y < 1\\ 0, & otherwise \end{cases}$$

(i) Find 
$$P(X < \frac{1}{2}, Y < \frac{1}{4})$$
.

(ii) Find all the marginal and conditional probability density functions.

[(Evaluate/HOCQ)]

(b) For the following transition probability matrix for states  $\{0, 1, 2, 3\}$ 

[0]	0.2	0.8	0 ]	
0.3	0.1	0	0.6 0.5	
0.5	0	0	0.5	
0	0	0	1	
0.3 0.5 0	0	Ū		

- (i) draw the state transition diagram
- (ii) identify the recurrent, transient and absorbing states, and the communicating classes. [(Evaluate/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	37.5	37.5	25.0

\*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question