DESIGN & ANALYSIS OF ALGORITHMS
(CSEN 2201)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Consider a weighted complete graph G on the vertex set $\{v_1, v_2, ..v_n\}$ such that the weight of the edge (v_i, v_j) is $2|i-j|$. The weight of a minimum spanning tree of G is
(a) n−1 (b) 2n−2 (c) n/2 (d) n^2.

(ii) Dijkstra's algorithm cannot be applied on
(a) directed and weighted graphs (b) graphs having negative weight function
(c) unweighted graphs (d) undirected and unweighted graphs.

(iii) What is the time complexity of counting sort for n number of elements where the maximum element is m?
(a) O(m + log n) (b) O(n + log m)
(c) O(log m log n) (d) None of the above.

(iv) Consider the following recurrence equation:

$$T(n) = \begin{cases} k & \text{for } n = 1 \\ 3T(n/2)+kn & \text{for } n>1 \end{cases}$$

Which of the following statements is TRUE?
(a) $T(n) = O(n^{1.59})$ (b) O(1) (c) O(nlog n) (d) None of these.

(v) The tightest lower bound on the number of comparisons, in the worst case, for comparison-based sorting is of the order of
(a) N (b) N^2 (c) NlogN (d) N(logN)^2.

(vi) Merge procedure in Merge sort takes
(a) O(nlogn) time (b) O(n) time
(c) O(1) time (d) none of the mentioned.

(vii) A priority queue is implemented as a Max-Heap. Initially, it has 5 elements. The level-order traversal of the heap is given below:
10, 8, 5, 3, 2
Two new elements '1' and '7' are inserted in the heap in that order. The level-order traversal of the heap after the insertion is:
(a) 10, 8, 7, 5, 3, 2, 1 (b) 10, 8, 7, 2, 3, 1, 5
(c)10, 8, 7, 1, 2, 3, 5 (d) 10, 8, 7, 3, 2, 1, 5.

(viii) Let G be a weighted connected undirected graph with distinct positive edge weights. If every edge weight is increased by the same value, then which of the following statements is/are TRUE?
P: Minimum spanning tree of G does not change
Q: Shortest path between any pair of vertices does not change
(a) P only (b) Q only (c) neither P nor Q (d) Both P and Q.

(ix) Let S be an NP-complete problem and Q and R be two other problems not known to be in NP. Q is polynomial time reducible to S and S is polynomial-time reducible to R. Which one of the following statements is true?
(a) R is NP-complete (b) R is NP-hard
(c) Q is NP-complete (d) Q is NP-hard.

(x) An array of 30 distinct elements is to be sorted using quicksort. Assume that the pivot element is chosen uniformly at random. The probability that the pivot element gets placed in the worst possible location in the first round of partitioning (rounded off to 2 decimal places) is _____
(a) 0.03 (b) 0.06 (c) 0.07 (d) 0.04.

Group- B

2. (a) Solve using Master theorem
$T(n) = 7T(n/2)+ n^2$. [(CSEN2201.4)(Analyze/IOCQ)]

(b) Show that a MAX-HEAP can be built in O(n) time from a given unordered array of n integers. You need not to write pseudo-code for building a MAX-HEAP. Give the time-complexity analysis only. [(CSEN2201.4)(Analyze/IOCQ)]

(c) Suppose a O(n) time deterministic algorithm is there to find the median of an unsorted array. Now, consider a QuickSort implementation where the median is first found using the above mentioned algorithm and then that median is used as pivot. Estimate the worst case time complexity of this modified QuickSort. Your answer should provide a tight bound. Justify your answer.
[(CSEN2201.4)(Analyze/IOCQ)]

(d) Order the following functions by growth rate: N, $N^{\sqrt{2}}$, N log N, N log log N, $N^{1.5}$, $N\log(N^2)$, 2/N, 2^N, $2^{N/2}$. Identify the functions with same growth rate (if any).
[(CSEN2201.2)(Understand/LOCQ)]
3 + 3 + (1 + 3) + 2 = 12

3. (a) Show that one can sort any 4 numbers given in an array in 5 comparisons.

Then use it (or otherwise) to show that 5 numbers can be sorted in 8 comparisons in the worst-case. [(CSEN2201.3)(Apply/IOCQ)]

(b) You are provided as input an even positive integer $n > 1$ and an unsorted sequence S of n integers. You would like to find *both* the second order statistic and the nth order statistic of S. Show that they can be found out in $(3n/2 + \lceil \lg n \rceil - 3)$ number of pair-wise comparisons. [(CSEN2201.4)(Analyze/IOCQ)]

(c) You have read a proof that the lower bound for comparison sort is $\Omega(n \lg n)$. But in the last semester you had been taught some sorting algorithm that ran in $O(n)$ time. What is your conclusion about this anomaly – there was a mistake in the analysis or something else? [(CSEN2201.2)(Understand/LOCQ)]

(2 + 2) + 6 + 2 = 12

Group - C

4. (a) Find out an optimal Huffman code for the following set of frequencies, based on the first 6 Fibonacci numbers. a:1 b:1 c:2 d:3 e:5 f:8 [(CSEN2201.3)(Apply/IOCQ)]

(b) (i) Fill up the spaces (Asymptotic values are fine) in the following table with respect to Prim's algorithm. [(CSEN2201.1)(Remember/LOCQ)]

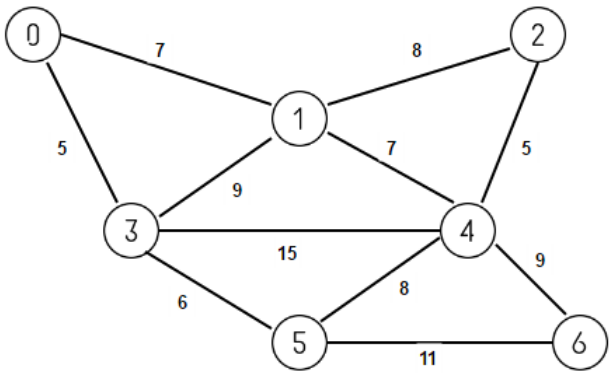
	How the Priority Queue will be implemented		
	Array	Binary Heap	Fibonacci Heap
Each Extract-Min will take			
Each Decrease-key will take			
Final Asymptotic Complexity			

(ii) Now from above you see that there are 3 different options for choosing the data structure to implement the Priority queue. Which option will turn out to be the best and which one will be the worst for the following 3 cases – (a) $E = O(V)$, (b) $E = O(V \log_2 V)$ and (c) $E = O(V^2)$. [(CSEN2201.5)(Evaluate/HOCQ)]

(iii) If there is a negative weighted edge in a given graph, will Bellman Ford always be able to give a solution? [(CSEN2201.1)(Remember/LOCQ)]

4 + (4.5 + 3 + 0.5) = 12

5. (a) Compute the Minimum Spanning Tree and its cost for the following graph using Kruskal's Algorithm implemented by disjoint set data structure. Indicate each step clearly. [(CSEN2201.3)(Apply/IOCQ)]



(b) Suppose you have n jobs to be performed by a robot. To perform each of them the robot requires one unit of time. At any point of time, the robot can do only one job. Moreover, if a job is started, it must be finished before starting the next job. For each i, there is a deadline $t_i \in \{0, 1, 2, \ldots, n-1\}$ associated with the i-th job. If the job is started at a time $\leq t_i$, the robot gets a profit of p_i (a positive integer). If the job is started at time $> t_i$, there is no profit at all. This means if a job cannot be scheduled on or before its starting deadline, the job need not to be scheduled at all. Notice that the deadlines may not be distinct from one another. You need to select and schedule a subset of the jobs for the robot in such a way that the robot's profit is maximized. Propose an $O(n \log n)$ time greedy algorithm to solve this problem. Analyze that your algorithm has the given running time. [(CSEN2201.6)(Design/HOCQ)]

6 + (4 + 2) = 12

Group - D

6. (a)
```
TABLE_INSERT(T, x)
1.  if size(T)=0
2.  then allocate tab[T] with 1 slot
3.  size[T]  ←  1
4.  if num[T] = size[T]
5.  then allocate new-table with 2. size[T] slots
6.  insert all old items to new-table
7.  free tab[T]
8.  tab[T] ← new-Table
9.  size[T]  ←  2. size[T]
10. insert x into tab[T]
11. num[T] ←  num[T]  + 1
```
Using accounting method, calculate the amortized cost of each TABLE_INSERT operation. [(CSEN2201.4)(Analyze/IOCQ)]

(b) Draw state-transition diagram for the string-matching automaton that accepts all strings ending in the string abbababbabbaba. [(CSEN2201.3)(Apply/IOCQ)]

(c) State the asymptotic time complexity of the KMP algorithm used for pattern matching and remember to define the terms you use. [(CSEN2201.1)(Remember/LOCQ)]

6 + 5 + 1 = 12

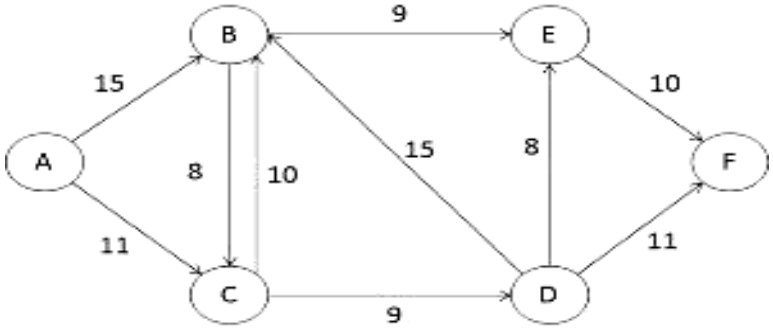
7. (a) Suppose we perform a sequence of n operations on a data structure in which the ith operation costs i if i is an exact power of 2, and 1 otherwise. Use aggregate analysis and accounting method to determine the amortized cost per operation. [(CSEN2201.3)(Apply/IOCQ)]

(b) Write a short note on skip list. Show that the expected space usage of a skip list with n items is $O(n)$. [(CSEN2201.2) (Understand/LOCQ)]

6 + (3 + 3) = 12

Group - E

8. (a)



Using Ford Fulkerson algorithm, find out the amount of maximum flow of the above flow network. [(CSEN2201.3)(Apply/IOCQ)]

(b) Say there are two disjoint sets: {1, 2, 100} and {5, 78, 90}.
Draw two diagrams to compare between linked list representation and rooted tree representation of the above two sets. Also show how these data structures will be modified after the UNION operation.
[(CSEN2201.2)(Understand/LOCQ)]

6 + (3 + 3) = 12

9. (a) Prove that the Clique Decision Problem is NP-hard by using the result that the 3-CNF-SAT problem is computationally hard. [(CSEN2201.5)(Assess/HOCQ)]

(b) (i) Fill in the gap:- "No _____ approximation algorithm exists for a general TSP." [(CSEN2201.1)(Remember/LOCQ)]

(ii) Give a 2-approximation algorithm for a TSP that satisfies triangle inequality and give a brief proof that it indeed maintains the approximation ratio.
[(CSEN2201.5)(Assess/HOCQ)]

6 + (0.5 + (3.5 + 2)) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	23.44	55.21	21.35

Course Outcome (CO):

After the completion of the course students will be able to

CSEN2201.1. Remember time complexities of various existing algorithms in different situations.

CSEN2201.2. Understand the basic principles of different paradigms of designing algorithms.

CSEN2201.3. Apply mathematical principles to solve various problems.

CSEN2201.4. Analyze the worst, best, and average case complexities of various algorithms.

CSEN2201.5. Assess the computational hardness of a problem and learn how some of the well-known problems are proved to be NP-hard and also design approximation algorithms for some of them.

CSEN2201.6. Create/Design a good algorithm for a new problem given to them.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question