COMPUTATIONAL FLUID DYNAMICS
(CHEN 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In a marching problem the differential equation is ______________
(a) parabolic (b) hyperbolic
(c) elliptic (d) normal ODE.

(ii) For a creeping flow, Navier-Stoke's equation will be reduced to ____________
(a) $\frac{Du}{Dt} = \mu\nabla^2 u$ (b) $\frac{Du}{Dt} = -\nabla p$
(c) $-\nabla p + \mu\nabla^2 u = 0$ (d) $\nabla p = 0$

(iii) For irrotational flow, the condition is given by ____________
(a) ∇.u=0 (b) u(∇.u)=0
(c) u(∇xu)=0 (d) ∇xu=0

(iv) Order of truncation in difference scheme provide an estimate of ________
(a) accuracy of the result
(b) the difference between the analytical and numerical values
(c) the rate of decrease of the error with grid size
(d) none of the above.

(v) Finite volume scheme relies on ______________
(a) the conservation form of the balance equations
(b) non-divergence form of the balance equations
(c) the continuity equation alone
(d) none of the above.

(vi) The forward difference scheme _________________
(a) requires three consecutive points for building derivatives
(b) requires two consecutive points for building derivatives
(c) requires alternate points for building derivatives
(d) uses the analytical derivative.

(vii) In a multigrid iteration scheme, the internode distance at level 3 of coarse grid is equal to ________________, when 'h' is the internode distance for fine grids.

(a) 2h (b) 8h
(c) 4h (d) 16h

(viii) The Lagrangian rate of change of property ϕ is given by ____________

(a) $\frac{D\phi}{Dt}$ (b) $\frac{\partial \phi}{\partial t}$
(c) $u.\nabla\phi$ (d) $\nabla^2\phi$

(ix) In hybrid scheme to find the value of property ϕ at face, central difference scheme is applied for ____________

(a) $Pe=2$ (b) $Pe>2$
(c) $Pe<2$ (d) $2<Pe<10$.

(x) Neuman boundary condition in heat transfer problem ________________

(a) applies when the boundary temperature is specified
(b) applies when the boundary temperature is held at zero
(c) applies when the heat flow through the boundary is specified
(d) applies when both the boundary temperature and the heat flow through the boundary are specified.

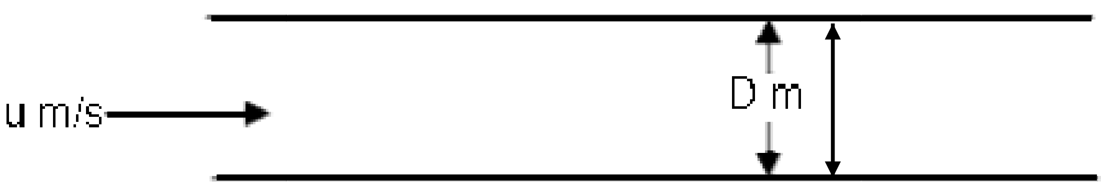
Group – B

2. (a) Show that the dimensionless form of the Navier-Stoke's equation can be written as, $St\frac{\partial U_i}{\partial \tau}+\sum\frac{\partial(U_iU_j)}{\partial X_j}=\frac{1}{Re}\sum\frac{\partial^2 U_i}{\partial X_j^2}-\frac{\partial P}{\partial X_i}$. The dimensionless notation for velocity, pressure, time and space U, P, τ and X respectively. [(CO1)(Understand/LOCQ)]

(b) "In the integral form of the transport equation for property ϕ, a term "$\int_A n\bullet(\Gamma grad\phi)dA$" represents net increase of the property due to inflow diffusional flux across the surface of the control volume." – Elaborate the correctness of the statement. [(CO1)(Understand/LOCQ)]

8 + 4 = 12

3.



Derive one convection-diffusion type equation for the one directional fluid flow between two parallel plates (as shown in the above figure) of width 'w' m and 'D' m apart. The velocity of the fluid at the leading edge is u m/s. The viscosity and density of the fluid are μ Pa.s and ρ kg/m^3 respectively. [(CO1)(Create/IOCQ)]

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Group – C

4. Show that the forward-in-time-backward-in-space (FTBS) scheme to solve a flow domain given by the equation $\frac{\partial\phi}{\partial t}+u\frac{\partial\phi}{\partial x}=|A|$ is conditionally stable. A is a constant. [(CO2)(Apply/IOCQ)] **12**

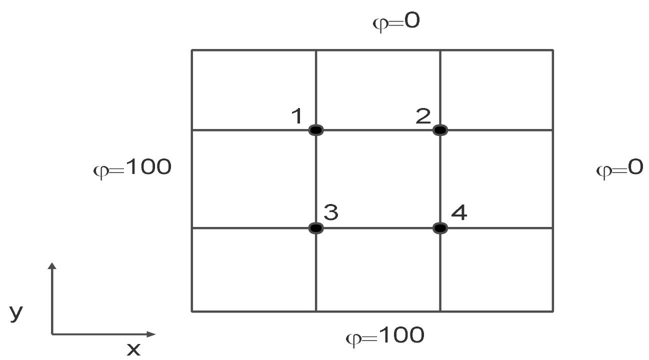
5. Heat is flowing in a rectangular slab of metal and can be modelled using the equation $\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$ at steady state. The left edge of the slab is maintained at T_H and the right edge is at T_C. All other edges are maintained at temperature, T_{amb}. Write down the general form of the algebraic equation after applying finite difference scheme to solve temperature vector '$\underline{T}$' in the form $\boldsymbol{A}\underline{T}=\underline{Q}$
Assuming 4 grid points in the x and y direction of the slab, show the exact form of the **A** matrix. [(CO2)(Apply/IOCQ)] **(6 + 6) = 12**

Group – D

6. For a convection-diffusion property transport problem in one direction the equation is given as $\frac{\partial\phi}{\partial t}=\Gamma\frac{\partial^2\phi}{\partial x^2}$. Find out the solution matrix for the system within a length of 1 m using central differencing, when Γ=0.1 kg/ms, u=2.5 m/s, ρ=1 kg/m^3. The boundary conditions are given as Φ(0)=1 and Φ(1)=0. Assume Δx=0.1. [(CO3)(Apply/IOCQ)] **12**

7. (a) "Hybrid scheme can only be applied to a staggered grid arrangement, but not to a collocated grid arrangement." – Comment on the correctness of the statement with adequate explanation. [(CO3)(Evaluate/HOCQ)]

(b)



From the above figure, where φ is governed by $\text{div}(\rho u\varphi)=\text{div}(\Gamma\,\text{grad}\varphi)+a-b\varphi$, where ρ=1, Γ=1, a=10 and b=2. Δx=Δy=1. The flow in the x-direction=u=1 and the flow in the y-direction=v=4. Using upwind scheme find out values for φ_1, φ_2, φ_3, and φ_4. [(CO3)(Apply/IOCQ)] **4 + 8 = 12**

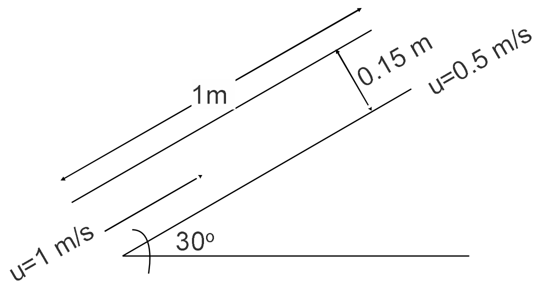
Group – E

8. (a) "For a pressure-velocity coupled flow field, the staggered grid conformation becomes useful instead of collocated grid." – Justify the appropriateness of the statement with relevant mathematical expression. [(CO4)(Evaluate/HOCQ)]

   (b) Elaborate the SIMPLE algorithm for a 2D Cartesian flow with adequate mathematical derivations. [(CO4)(Remember/LOCQ)]

3 + 9 = 12

9.



A liquid of density 1000 kg/m^3 and viscosity 0.001 Pa.s is flowing over the inclined plane as shown in the above figure. Using artificial compressibility method, develop a solution matrix for evaluating pressure and velocity at three intermediate nodes. Assume the flow is in streamlined condition. Given: Sound velocity=1520 m/s. [(CO4)(Apply/IOCQ)]

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