MATHEMATICAL METHODS (MATH 2001)

Time Allotted : 3 hrs. **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For the function $f(z) = |z|^2$, $f(z)$ is
(a) analytic everywhere. (b) analytic at $z = 0$.
(c) analytic nowhere. (d) analytic when $z \neq 0$.

(ii) The regular singular point(s) of the ordinary differential equation $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = 0$ is/are
(a) 1,0 (b) only 1 (c) $1, -1$ (d) only -1.

(iii) The value of $\oint_C \frac{dz}{z-3}$ where $C: |z| = 1$ is
(a) $-3i$. (b) $-3\pi i$. (c) $3\pi i$. (d) 0.

(iv) One dimensional wave equation is of the form
(a) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. (b) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.
(c) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$. (d) $\frac{\partial u}{\partial t} = c \frac{\partial u}{\partial x}$.
where $u(x, t)$ is a function of x and t and c is a constant.

(v) The function $f(z) = \frac{e^{z^2}}{z^4}$ has
(a) essential singularity at $z = 0$. (b) a pole of order 4 at $z = 0$.
(c) a simple pole at $z = 0$. (d) removable singularity at $z = 0$.

(vi) The period of $f(x) = 2|cos^2 x|$ is
(a) 2π. (b) $\frac{\pi}{2}$. (c) 3π. (d) π.

(vii) A function $f(x), a < x < b$, can be expanded in a Fourier series
(a) only if it is continuous everywhere
(b) even it is discontinuous at a finite number of points in (a, b)
(c) even it is unbounded in (a, b)
(d) only if it is both continuous and bounded in (a, b).

(viii) Bessel's equation of order zero is

(a) $xy'' + y' - xy = 0$. (b) $xy'' + y' = 0$.

(c) $xy'' - y' + xy = 0$. (d) $xy'' + y' + xy = 0$.

(ix) Given $\left(\frac{\partial^2 z}{\partial x^2}\right)^3 + \left(\frac{\partial z}{\partial x}\right)^4 = \frac{\partial z}{\partial y} + 2$, then which of the following statement is correct?

(a) The equation is of order 2, degree 4, linear.
(b) The equation is of order 1, degree 3, non-linear.
(c) The equation is of order 2, degree 3, linear.
(d) The equation is of order 2, degree 3, non-linear.

(x) The value of $J_{\frac{1}{2}}^2(x) + J_{-\frac{1}{2}}^2(x) =$

(a) -1. (b) 0. (c) $\frac{2}{\pi x}$. (d) $\frac{2}{\pi x} sin 2x$.

Group – B

2. (a) (i) Prove that $\lim_{z\to 0} \frac{\bar{z}}{z}$ does not exist.

[(MATH2001.1, MATH2001.2) (Understand /LOCQ)]

(ii) Show that $f(z) = xy + iy$ is not analytic anywhere.

[(MATH2001.1, MATH2001.2) (Apply/IOCQ)]

(b) Evaluate $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ for (i) $C: |z| = 1$, (ii) $C: |z| = \frac{1}{4}$.

[(MATH2001.1, MATH2001.2) (Apply/IOCQ)]

(4 + 2) + 6 = 12

3. (a) Use Cauchy's residue theorem to evaluate $\oint_C \frac{z^2+z-1}{(z^2-1)(z-3)} dz$ around the circle $C: |z| = 2$. [(MATH2001.1/MATH2001.2) (Evaluate/HOCQ)]

(b) Determine the analytic function whose imaginary part is $v(x, y) = e^x \sin y$.

[(MATH2001.1/MATH2001.2) (Apply/IOCQ)]

6 + 6 = 12

Group – C

4. (a) Obtain a Fourier series expansion for $f(x) = x + x^2$ in $-\pi < x < \pi$. Hence show that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$

[(MATH2001.1, MATH2001.3, MATH2001.4) (Remember/LOCQ)]

(b) Evaluate (i) $F\left(4x^2 e^{-3|x|}\right)$ (ii) $F\left(e^{-2x^2} \cos 3x\right)$.

[(MATH2001.1, MATH2001.3, MATH2001.4) (Understand/LOCQ)]

6 + 6 = 12

5. (a) Express $f(x) = x$ in a half range cosine series in $0 < x < l$, and using Parseval's Identity deduce the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots$ ... .

[(MATH2001.1/MATH2001.3/MATH2001.4) (Apply/IOCQ)]

(b) Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} a - |x|, & for\ |x| < a \\ 0\ , & for\ |x| > a \end{cases}$ and hence show that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.
[(MATH2001.1, MATH2001.3, MATH2001.4) (Evaluate/HOCQ)]

6 + 6 = 12

Group – D

6. (a) Show that $x = 0$ is an ordinary point of $(x^2 - 1)y'' + 3xy' + xy = 0$ and also find the series solution for the differential equation about $x = 0$.
[(MATH2001.5) (Evaluate/HOCQ)]

(b) Express $2x^3 + x^2 + 1$ in terms of Legendre polynomials $P_0(x), P_1(x), P_2(x)$ and $P_3(x)$. [(MATH2001.5) (Remember/LOCQ)]

8 + 4 = 12

7. (a) Prove that $(2n + 1)xP_n(x) = (n + 1)P_{n+1}(x) + nP_{n-1}(x)$.
[(MATH2001.5) (Apply/IOCQ)]

(b) Prove that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$. [(MATH2001.5) (Apply/IOCQ)]

6 + 6 = 12

Group – E

8. (a) Construct a partial differential equation by eliminating the arbitrary functions $f(x)$and $g(x)$ from the relation
$z = yf(x) + xg(y)$ where $f(x)$ and $g(y)$ are functions of x and y alone.
[(MATH2001.1/MATH2001.6) (Create/HOCQ)]

(b) Find the general solution of the following partial differential equation
$(y + zx)p - (x + yz)q = x^2 - y^2$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.
[(MATH2001.1/MATH2001.6) (Apply/IOCQ)]

6 + 6 = 12

9. (a) Solve the following equation by applying Charpit's method: $q = xp + p^2$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. [(MATH2001.1/MATH2001.6) (Apply/IOCQ)]

(b) Solve: $(D^2 - 3DD' + 2D'^2)z = (x + y)^2$ where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$.
[(MATH2001.1, MATH2001.6)(Apply/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	20.84%	52.08 %	27.08 %

Course Outcome (CO):

After the completion of the course students will be able to
MATH2001.1 Construct appropriate mathematical models of physical systems.
MATH2001.2 Recognize the concepts of complex integration, Poles and Residuals in the stability analysis of engineering problems.
MATH2001.3 Generate the complex exponential Fourier series of a function and make out how the complex Fourier coefficients are related to the Fourier cosine and sine coefficients.
MATH2001.4 Interpret the nature of a physical phenomena when the domain is shifted by Fourier Transform e.g. continuous time signals and systems.
MATH2001.5 Develop computational understanding of second order differential equations with analytic coefficients along with Bessel and Legendre differential equations with their corresponding recurrence relations.
MATH2001.6 Master how partial differentials equations can serve as models for physical processes such as vibrations, heat transfer etc.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question