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Article *in* Computers & Industrial Engineering · May 2016

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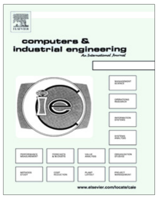
Computers & Industrial Engineering xxx (2016) xxx–xxx

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie



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Joint decision on EOQ and pricing strategy of a dual channel of mixed retail and e-tail comprising of single manufacturer and retailer under stochastic demand

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ARTICLE INFO

Article history:
Available online xxxx

Keywords and phrases:
Two-echelon supply chain
Dual-channel competition
Pricing
News vendor

ABSTRACT

The paper formulates a dual channel model for a two-echelon supply chain comprising of one manufacturer and one retailer for trading a single product. The manufacturer uses direct online (e-tail) channel and traditional (Brick and Mortar) retail channel to boost sell the products. A single-period news vendor type demand in the cases of integrated and Stackelbarg game approach is analyzed to obtain optimal stock level, sales prices, promotional effort and service level for both the e-tail and retail channel, and hence retailer competes with the manufacturer's direct channel. Finally, computational results show that dual channels influence significantly the pricing strategies and effort levels of the supply chain entities, and it is always beneficial in integrated system for the members of the chain.

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1. Introduction

In today's rapid changing marketing environment, the end customers have broken all barriers of the stereotype traditional purchasing habits. Long research and market surveys on the customers' purchasing habit suggest that today's tech savvy young generation loves to linger on a rigorous research of the product on the internet in which they desire to buy just sitting in their office cubicle and without putting any extra physical effort for that matter. As there are numerous option of buying a product online rather than going physically to the store burning fuel as well as lots off energy, the buyers prefer e-tail option for valid and self explanatory reason. This tendency has increased a lot of late. They prefer to buy the product via the Internet and sit back relax at home to wait for the product to arrive at their door step rather than going to a traditional retail shop escaping all the hazards. This rapid change has come into picture as now the customer can acquire his/her desired product through several ways other than the traditional retail channel, such as direct channel or a dual channel (a combination of traditional and direct selling channels). In a traditional retail channel, retailers buy products from the manufacturer at a wholesale price and sell them to the end customers. Whereas, in a direct channel, customers can purchase their desired products directly from manufacturers. This sometimes saves their money too as the price offered by the manufacturer directly is obviously bit less than the price offered by the retailer. On the other hand, in a dual channel, the consumer has the option of buying via a retail channel or a direct channel at the same time.

Under the above mentioned scenario sticking only to the traditional retail channel might not remain any more a good option for manufacturers. Its high time for a manufacturer to decide whether to stick to the traditional channel only or think for dual channel for selling their products. Manufacturers might have to face consequences of losing a great amount of market share in the given circumstances that few other manufacturers of product have already opened e-tail channel for selling their product and attracted potential customers. Consequently, the manufacturer who cannot cope with the changing nature of the market have to loss a great amount of profit. On the other hand, dual channel might increase manufacturer's profit as implementation of a dual channel gives more power to the manufacturer who can gain from selling via both channels. Hence, a dual channel can be proved to be very fruitful to increase the profit of the manufacturer. So, its important for the manufacturer to look for other options to sustain themselves in the market such as dual channel operation.

Now, how the dual channel operates? In a dual-channel situation, a manufacturer, apart from retail channels, sells directly to end customers by using either an online shop or through some

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http://dx.doi.org/10.1016/j.cie.2016.05.002
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Please cite this article in press as: Roy, A., et al. Joint decision on EOQ and pricing strategy of a dual channel of mixed retail and e-tail comprising of single manufacturer and retailer under stochastic demand. *Computers & Industrial Engineering* (2016), http://dx.doi.org/10.1016/j.cie.2016.05.002

third party vendor of online shopping such as “flipkart”, “snapdeal”, “amazon”, etc. Generally, a fear plays in the mind of the traditional and old-fashioned customer during buying the product from online channel. The fear is of loosing the quality, since they cannot see or feel the product quality physically by touching which is quite logical. So, to break this barrier of fear of buying products online, manufacturer has to act cleverly. They must entrust the customers by giving assurance of money back policy if quality of the product delivered to the customer does not satisfy their expectation. Also, to attract the customers, manufacturer offers after sales service level assurance/agreement (SLA) at the time of selling the product. From the end customers perspective, this enables them with the trust that they are buying the product directly from the manufacturer. SLA is an agreement between a service provider and a customer that specifies what services the service provider will furnish. There are some key parameters of SLA such as quality assurance, return material authorization, delivery authorization, voice of the customer feedback, among others. Many manufacturers act as service providers who provide their customers with a specific SLA. Quite often, departments in major companies have adopted the idea of service level agreement so that services for their customers can be measured, justified, and perhaps compared with outsourcing network providers. The price will also appear to be bit less than what they have to pay if they opt to buy the same product from any other retail shop. Obviously, this can really be a potential threat to the retailer as it can shrink the retailers market and threatens putting them out of business. So, traditional retailers have to employ some potential efforts as well as attractive gifts by lottery, extended warranty, advertising, etc., to regain the ground lost. Using proper promotional efforts, traditional retailers can bring back potential thriving customers who might otherwise be drawn towards the direct channel of the manufacturer. This promotional and sales efforts give retailer a tool to compete with the manufacturer’s direct channel and place them in a better trading position. In general, sales and promotional efforts made by retailer is supposed to boost individual demand as well as the demand of manufacturer’s direct-channel.

Several studies have been conducted by researchers considering dual-channel competition.In this direction, the work of Hotelling (1929) and Salop (1979) consider a circular spatial market model. Essentially, Salop (1979) has pioneered and introduced a variant of the traditional model which is the very early work of Hotelling (1929), known as the circular spatial market model, where the consumers are modeled to be uniformly distributed on a circle and all the firms are located evenly on the circle. Balasubramanian (1998) considers the competition between a direct marketer and conventional retailers in price. Dewan, Jing, and Seidmann (2003) develop a duopoly model where each firm first chooses a customization scope and then optimally determines the conventional market price.

Many firms have engaged in online channels to reach customers who cannot be reached by the traditional retail channel. It is reported that about 42 percent of the top suppliers such as IBM, Nike, Pioneer Electronics, Estee Lauder, and Dell are selling to customers through an online channel (Chiang, Chhajed, & Hess, 2003; Tsay & Agrawal, 2004). An important area related to the dual channel supply chain system is channel competition among the members. Paralar and Wang (1993) also analyze the case where independent firm demands are aggregated to form industry demand. Many researches focus on the effect of introducing direct Internet channels to supply chains as well as the existing retail channel and price-related issues under that circumstances. Choi (1996) considers a channel structure in which there are duopoly manufacturers and duopoly common retailers. Lippmand and Macardle (1997) study a competitive version of the classical newsboy problem in which a firm must choose an inventory or production level for perishable goods with random demand. Lal and Sarvary (1999) introduce an internet channel so as to reduce the price competition as well as consumer’s extra cost. It is observed that in hybrid channel system, the consumers are categorized into two groups: price sensitive and service sensitive. It has also been established that the manufacturer will not introduce an Internet channel to avoid price competition if there are considerate amount of service sensitive consumers. Swaminathan and Tayur (2003) discuss the opportunities and changes induced by Internet usage in supply chain management (SCM). Park and Keh (2003) use game theory to examine the equilibrium under hybrid channel system, particularly in terms of price and profit distribution. Yao, Yue, Wang, and Liu (2005) investigate an optimal order quantity of the retailer and buy-back price of the manufacturer when the direct channel is introduced. Mukhopadhyay, Zhu, and Yue (2008) show how, by means of an online channel, firms can deal with customers’ orders, and control the distribution and pricing of goods. Cai, Zhang, and Zhang (2009) evaluate the impact of price discount contracts and pricing schemes on the dual-channel supply chain competition. Meanwhile, from supplier-Stackelberg, retailer-Stackelberg, and Nash game theoretic perspectives, they show that the scenarios with price discount contracts can outperform the non-contract scenarios. Cai (2010) investigates the influence of channel structures and channel coordination on the supplier, the retailer, and the entire supply chain in the context of two single channel and two dual-channel supply chains. Lu, Tsao, and Charoensiriwath (2011) highlight the importance of service from manufacturers in the interactions between two competing manufacturers and their common retailer, facing end consumers who are sensitive to both retail price and manufacturer service, and a game-theoretic framework is applied to obtain the equilibrium solutions for each entity.

In dual channels business model, the manufacturer has two revenue sources, which would increase the manufacturer’s profit. However, the reservation of price of the direct channel over the indirect channel has some disadvantages. It is also potential reason of channel conflict between the two channels. Due to this reason, manufacturer’s profit may be reduced. Specially, when the retailer in the traditional retail channel is a Stackelberg leader of the supply chain, the counter-force for the development of the direct channel and channel conflict can be large. As a consequence, it becomes vague whether the manufacturer in a retailer-Stackelberg (RS) supply chain should use the dual channel strategy.In the literature, in view of the above mentioned scenarios, a number of studies have examined the RS supply models namely Gerchak and Wang (2004), Wang and Liu (2007), Pan, Lai, Leung, and Xiao (2010), Edirisinghe, Bichescu, and Shi (2011) and Choi and Fredj (2013). Few researchers have worked in this area to examine the effect of channel leadership structure. For example, Pan et al. (2010) discuss different contract strategies under both manufacturer-Stackelberg (MS) and retailer-Stackelberg (RS) supply chains and identify the conditions under which the leader is better off using a revenue-sharing contract. The channel leadership structure (sequence of decisions) affects the equilibrium outcome (Choi & Fredj, 2013) which further influences the channel structure strategy of the manufacturer. Edirisinghe et al. (2011) study the implications of channel power on supply chain stability for a two-supplier and one-retailer supply chain. Choi, Li, and Xu (2013) examine the closed loop supply chain with different channel leadership. In manufacturer-retailer supply chain systems, the research works of Panda (2013, 2014), Shah, Gor, and Jhaveri (2012), Shah and Shukla (2010), Cardenas-Barron, Taleizadeh, and Trevino-Garza (2012), Chung, Cardenas-Barron, and Ting (2014), Garcia-Laguna, San-José, Cardenas-Barron, and Sicilia (2010), Sarkar (2015) and Sarkar and Sarkar (2013) are worth mentioning, among others.

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Over the past decade, the dual-channel supply chain has gained much attention of the supply chain management research community. Chiang et al. (2003) examine a price-setting game between a manufacturer and a retailer in a dual channel based on the consumer choice model. This study shows that the manufacturer is more profitable even if no sales occur in the direct channel. Tsay and Agrawal (2004) provide a comprehensive review of quantitative approaches in multi-channel distribution systems that may coordinate the actions of channel partners. Cattani, Gilland, and Swaminathan (2004) do research on coordination opportunities that arise for firms having the dual channel. Cattani et al. (2006) and Huang and Swaminathan (2009) investigate the pricing decisions of the manufacturer and its retailers. Chen, Kaya, and Ozer (2008) discuss on service competition in the dual-channel supply chain. They find out that the manufacturer's optimal channel strategy depends on the channel environment. Zhang et al. (2012) propose the effect of product substitutability and relative channel status on pricing decisions under different power structures. Huang, Yang, and Liu (2013) observe the effect of production cost disruption in a dual-channel supply chain model. Ren, He, and Song (2014) study price and service competition in a dual-channel supply chain with consumer returns. Cao (2014) coordinates a dual-channel supply chain under demand disruption. Vinhas and Heide (2015) analyze the forms of competition and outcomes in a dual distribution channel.

In the present article, a two layer supply chain between manufacturer and retailer is studied in dual channels (e-tail and retail) systems. Under uncertainty of demand of the products, the manufacturer implements direct online (e-tail) channel along with retail channel and compares the best strategies adopted by the channel members in the dual systems. To attract the customers more, promotional effort and service level assurance are offered by the channel members. Consequently, uncertain demand is a function of e-tail price, retail price, promotional effort and service level assurance which is quite new formula compared to the existing literature in dual channel system. Finally, the expected profit functions in different scenarios are formulated and analyzed mathematically, Fortunately, the optimal values of the decision variables are explicitly obtained in this paper. As far as the knowledge of the authors goes, such type of dual channel model for uncertain demand involving six decision variables (lotsize of retail channel, lotsize of e-tail channel, e-tail price, retail price, promotional effort and service level assurance) has not yet been discussed in inventory literature.

The rest of the paper is organized as follows: Section 2 explains the fundamental notations and assumptions, Section 3 provides mathematical formulations and analysis of the model, In Section 4, numerical example is provided. Section 5 draws conclusion on the findings of the paper.

2. Fundamental assumptions and notation

The following assumptions are made to develop the model:

2.1. Assumptions

(i) The manufacturer produces a single product and sells through a direct channel and a retail channel.
(ii) The wholesale price of the product at the manufacturer's end must be less than the price of the product in the direct channel; otherwise, the retailer might tend to buy the product directly from the direct channel and sell it to the customers.
(iii) Manufacturers use after sales service level agreement to increase market demand for the direct e-tail channel.
(iv) Retailer uses promotional effort to boost the market demand of retail-channel. Manufacturer also share some portion of the promotional effort, as retailer's promotional efforts boost their individual demands as well as the manufacturer's direct-channel demand.
(v) The Demand rate of the members of the chain is assumed to be uncertain and price, service level and promotional effort sensitive.
(vi) The chain is with buyback policy.
(vii) The lead time is negligible.
(viii) Shortage at the retailer is permitted due to uncertain demand.
(ix) Replenishment rate is instantaneously infinite but it's size is finite.
(x) All members of the chain are risk-neutral and seek to maximize their own expected profit.
(xi) The cost of manufacturer's service level assurance is $\frac{ns^2}{2}$ which is strictly convex in the sales service level parameter s.
(xii) the cost of retailer's promotional effort is $k\rho^2$ which is also strictly convex in the promotional effort parameter ρ. As the retailer's promotional efforts provides benefit to both the retailer and the manufacturer, the manufacturer bears a portion t of the effort cost and the retailer shares the rest.

2.2. Notation

Q_r	Retailer's Order Quantity (unit) for the traditional retail channel.
Q_e	Manufacturer's inventory (unit) for the direct e-tail channel.
Q	Manufacturer's total inventory (unit), $Q = Q_r + Q_e$, for both the channel.
x	A part of demand quantity (units/month) during a period, which is a random variable following probability distribution.
$f(x)$	Probability density function of x.
$F(x)$	Cumulative distribution function of x.
$F^{-1}(x)$	Inverse function of F.
p_e	Unit e-tail price ($/unit) for manufacturer to sell in direct e-tail channel.
p_r	Unit retail price ($/unit) for retailer to sell through traditional retail channel.
$D(p_e)$	Demand (units/month) in direct e-tail channel which is a function of the e-tail price p_e.
$D(p_r)$	Demand (units/month) in traditional retail channel which is a function of the retail price p_r.
$D(p_e, p_r)$	Resulting demand (units/month) in dual channel environment under competition which is a function of the both e-tail price p_e and retail price p_r.
w	Wholesale price ($/unit) per unit of the manufacturer to the retailer (unit purchasing cost of products at the retailer) in the traditional retail channel.
c_e	Purchasing cost/procurement cost ($/unit) of the manufacturer in the direct e-tail channel.
c_r	Purchasing cost/procurement cost ($/unit) of the manufacturer in the traditional retail channel.
v	Unit salvage value/return price ($/unit) under buy back contract of unsold goods of the retailer provided by the manufacturer under the traditional retail channel.
r	Shortage cost ($/unit) of the retailer.

(continued on next page)

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r_e	Shortage cost ($/unit) of the manufacturer in the direct e-tail channel.
s	Manufacturers service level bore towards the customer for the direct e-tail channel.
ρ	Denotes the promotional/advertising effect bore by the retailer towards the customer for the retail channel.
α	Denotes the demand sensitivity of the end customer towards the promotional/advertising effect for the retail channel.
β_e	Denotes the demand sensitivity of manufacturer on its own e-tail price p_e.
β_r	Denotes the demand sensitivity of retailer on its own retail price p_r.
γ_e	Denotes the demand sensitivity of the retailer on manufacturer's price under dual channel.
γ_r	Denotes the demand sensitivity of the manufacturer on retailer's price under dual channel.
δ	Denotes the demand sensitivity of end customers towards the after sale service assurance provided by the manufacture for the e-tail channel.
t	Denotes the fraction of expenditure incurred for promotional effort shared by the manufacturer.
μ	Denotes the mean demand of the retailer.
E_{mr}	Expected profit ($/month) function of the manufacturer under the traditional retail channel.
E_{rr}	Expected profit ($/month) function of the retailer under the traditional retail channel.
E_{ec}	Expected profit ($/month) function of the manufacturer under the direct e-tail channel.
EIP_{dc}	Expected integrated profit ($/month) function of the channel under the dual channel.
x^+	$Max[x, 0]$, the positive part of x.

3. Mathematical formulation and analysis of the model

A single manufacturer's channel choice decision for a single product is examined in this model. In this context, the following three choices (Fig. 1) are considered to formulate the model: (a) the traditional retail (bricks and mortar) channel or RC indexed as r; or (b) the direct online channel or EC indexed as e; and (c) both the RC and EC or a DC (dual channel) and indexed as dc. The RC represents the established configuration where the manufacturer sold its product to a retailer and then retailer displays its products in a traditional retail store and customers are required to travel to the store location to purchase the product. These types of structures are well established in practice and the cost per unit to process a product through this channel is assumed to be c_r. An 'online' channel (EC) could be introduced as a replacement for the current channel or as an additional channel for serving the market demand and, in this setting, the manufacturer incurs a per unit cost c_e for processing the product through this channel. The retailer and the manufacturer are two risk-neutral firms. The retailer incurs some promotional effort cost to increase brand consciousness on the local market. The manufacturer negotiates with the retailer and fixes a sharing policy for the promotional effort cost assuming that total demand of the manufacturer would be increased indirectly. Consequently, market demand is influenced by the advertising expenditure incurred by the retailer that results in promoting the product. Further, this promotional effort affects basic demand in such a way that demand rate becomes an increasing function of the cost of promotional effort. Here, ρ is a decision variable associated with the effort for promotional activities. On the other hand, e-tail channel customers are acquainted with the product through the electronic media, which already has a promotional background due to the retail channels' investment of the brand endorsement. So, an EC would no more require any further expenditure in the promotional activity, rather what would be beneficial for the EC is the sales service assurance (service level agreement) to the users. As a result, service level assurance for this e-tail channel is considered as s. Therefore, the demand of the products is assumed to be the function of e-tail price p_e, p_r, promotional effort $\rho$and service assurance s. The functional form of demands D_r in RC and D_e in EC are as follows:

$$D_r = x - \beta_r p_r + \alpha\rho \tag{1}$$

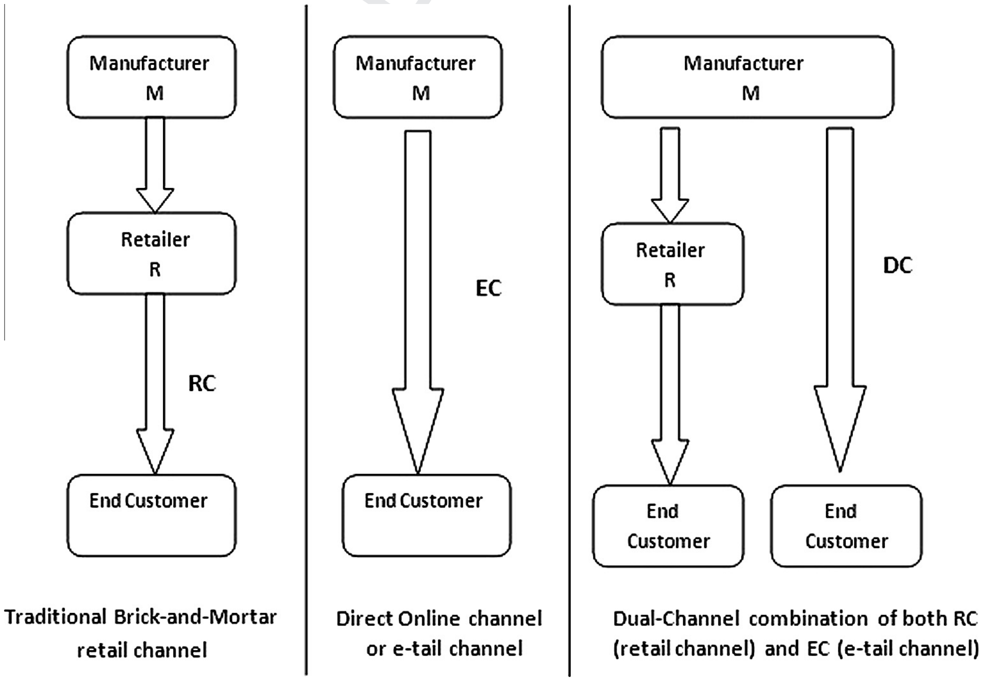

Fig. 1. Schematic diagram of the dual channel system.

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and

$$D_e = x - \beta_e p_e + \delta s. \tag{2}$$

In dual channel (DC) system, we can have two different demands considering the fact that there is a competition between two channels while performing together. Demand for the manufacturer's traditional RC is D_{rr} and demand for the retailer's direct online EC is D_{ee} which are defined as follows:

$$D_{rr} = x - \beta_r p_r + \gamma_e p_e + \alpha\rho \tag{3}$$

and

$$D_{ee} = x - \beta_e p_e + \gamma_r p_r + \delta s. \tag{4}$$

Here, in the dual channel environment, the demand function is decreasing in its own channel's retail price and increasing in the competing channel's retail price. The parameters β, γ measure the responsiveness of market demand to its own channel's retail price and competing channel's retail price, respectively. The random variable x describes the base-case potential market size for the product that follows p.d.f $f(x)$, i.e., $\int_0^\infty f(x)dx = 1$. As the demand is uncertain, there is a buyback policy between the manufacturer and the retailers in the traditional retail channel (RC). Suppose the retailer purchase Q_r quantity of lot size with wholesale price w from the manufacturer and failed to sell all of them by the end of the season then the unsold items at retailers are buyback to the manufacturer at a salvage value υ. On the other hand, shortages at the manufacturer may occur due to direct online e-tail channel (EC) as there the products are directly sold to the end customers. Thats why, a shortage cost r_e in the e-tail channel (EC) is incorporated. In this situation, the expected profits under the different channel environments are given as follows:

$$E_{mr}[Q_r, \rho] = (w - c_r)Q_r - \upsilon E(A - x)^+ - tk\rho^2, \tag{5}$$

$$\begin{aligned} E_{rr}[p_r, \rho] &= p_r E(D_r) - wQ_r + \upsilon E(A - x)^+ - rE(x - A)^+ - (1 - t)k\rho^2 \\ &= p_r(\mu - \beta_r p_r + \alpha\rho) - wQ_r + \upsilon E(A - x)^+ - rE(x - A)^+ \\ &\quad - (1 - t)k\rho^2, \end{aligned} \tag{6}$$

Therefore, the expected profit of the retail channel is

$$\begin{aligned} E_{rc}[Q_r, p_r, \rho] &= E_{mr} + E_{rr} \\ &= -Q_r c_r + p_r(\mu - \beta_r p_r + \alpha\rho) - rE(x - A)^+ - k\rho^2, \end{aligned} \tag{7}$$

where

$$A = (Q_r + \beta_r p_r - \alpha\rho),$$

$$E(A - x)^+ = \int_0^A (A - x)f(x)dx,$$

$$E(x - A)^+ = \int_A^\infty (x - A)f(x)dx$$

and

$$F(A) = \int_0^A f(x)dx.$$

In EC system, the expected profit of the chain is

$$\begin{aligned} E_{ec}[Q_e, p_e, s] &= p_e E(D_e) - Q_e c_e - r_e E(x - B)^+ - \frac{ns^2}{2} \\ &= p_e(\mu - \beta_e p_e + \delta s) - Q_e c_e - r_e E(x - B)^+ - \frac{ns^2}{2}, \end{aligned} \tag{8}$$

where

$$B = (Q_e + \beta_e p_e - \delta s),$$

$$E(x - B)^+ = \int_B^\infty (x - B)f(x)dx$$

and

$$F(B) = \int_0^B f(x)dx.$$

In DC environment, using the demand under competition of the dual chain, the individual expected profits of the chain are as follows:

$$E_{mr}[Q_r, \rho] = (w - c_r)Q_r - \upsilon E(D - x)^+ - tk\rho^2 \tag{9}$$

and

$$E_{rr}[p_r, \rho] = p_r E(D_{rr}) - wQ_r + \upsilon E(D - x)^+ - rE(x - D)^+ - (1 - t)k\rho^2, \tag{10}$$

where

$$D = (Q_r + \beta_r p_r - \gamma_e p_e - \alpha\rho),$$

$$E(D - x)^+ = \int_0^D (D - x)f(x)dx,$$

$$E(x - D)^+ = \int_D^\infty (x - D)f(x)dx$$

and

$$F(D) = \int_0^D f(x)dx.$$

Therefore, expected profit of the retail channel is

$$\begin{aligned} E_{rc}[Q_r, p_r, \rho] &= E_{mr} + E_{rr} \\ &= -Q_r c_r + p_r(\mu - \beta_r p_r + \gamma_e p_e + \alpha\rho) \\ &\quad - rE(x - D)^+ - k\rho^2 \end{aligned} \tag{11}$$

and expected profit of the e-tail channel is

$$\begin{aligned} E_{ec}[Q_e, p_e, s] &= p_e E(D_{ec}) - Q_e c_e - r_e E(x - G)^+ - \frac{ns^2}{2} \\ &= -Q_e c_e + p_e(\mu - \beta_e p_e + \gamma_r p_r + \delta s) - r_e E(x - G)^+ - \frac{ns^2}{2}, \end{aligned} \tag{12}$$

where

$$G = (Q_e + \beta_e p_e - \gamma_r p_r - \delta s),$$

$$E(x - G)^+ = \int_E^\infty (x - G)f(x)dx$$

and

$$F(G) = \int_0^E f(x)dx.$$

As a whole, the expected profit of the integrated channels is

$$\begin{aligned} EIP_{dc}[Q_r, Q_e, p_r, p_e, \rho, s] = E_{rc} + E_{ec} &= -Q_r c_r - Q_e c_e \\ &\quad + p_r(\mu - \beta_r p_r + \gamma_e p_e + \alpha\rho) \\ &\quad + p_e(\mu - \beta_e p_e + \gamma_r p_r + \delta s) - rE(x - D)^+ \\ &\quad - r_e E(x - G)^+ - k\rho^2 - \frac{ns^2}{2}. \end{aligned} \tag{13}$$

Here, the maximization strategies are to consider the following maximization problems:

- RC: *Maximize* $\mathbf{E_{rc}}$ with respect to decision variables $\{Q_r, p_r, \rho\}$.
- EC: *Maximize* $\mathbf{E_{ec}}$ with respect to decision variables $\{Q_e, p_e, s\}$.

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- DC: *Maximize* $\mathbf{EIP_{dc}}$ with respect to decision variables $\{Q_r, Q_e, p_r, p_e, \rho, s\}$.

Now, the main objective is to determine the optimal values of the economic order quantity (EOQ), the sales prices, promotional effort and service level assurance so that the profit of the channel is maximized. The above maximization problems would be discussed in the next sections.

3.1. Traditional Retail Channel environment (RC)

For each single channel environment, the working procedure of obtaining an optimal solution to the channel member's profit maximization problem is similar. Now, the retail channel under both the centralized and decentralized systems would be analyzed as follows:

3.1.1. Case-I: Centralized Supply chain (CS)

In this system, both the manufacturer and the retailer make decision jointly. Then, the prime objective of the members of the chain is to maximize the integrated expected profit of the system. The expected integrated profit of the channel is

$$E_{rc}[Q_r, p_r, \rho] = -Q_r c_r + p_r(\mu - \beta_r p_r + \alpha\rho) - rE(x-A)^+ - k\rho^2.$$

The partial derivatives of E_{rc} with respect to Q_r, ρ and p_r are as follows:

$$\frac{\partial E_{rc}}{\partial Q_r} = (r - c_r) - rF(A), \tag{14}$$

$$\frac{\partial^2 E_{rc}}{\partial Q_r^2} = -rf(A) < 0, \tag{15}$$

$$\frac{\partial E_{rc}}{\partial p_r} = (\mu - 2\beta_r p_r + \beta_r r + \alpha\rho) - r\beta_r F(A), \tag{16}$$

$$\frac{\partial^2 E_{rc}}{\partial p_r^2} = -2\beta_r - r\beta_r^2 f(A) < 0, \tag{17}$$

$$\frac{\partial E_{rc}}{\partial \rho} = (\alpha p_r - 2k\rho - \alpha r) + r\alpha F(A), \tag{18}$$

$$\frac{\partial^2 E_{rc}}{\partial \rho^2} = -2k - r\alpha^2 f(A) < 0, \tag{19}$$

$$\frac{\partial^2 E_{rc}}{\partial Q_r \partial \rho} = \frac{\partial^2 E_{rc}}{\partial \rho \partial Q_r} = r\alpha f(A), \tag{20}$$

$$\frac{\partial^2 E_{rc}}{\partial Q_r \partial p_r} = \frac{\partial^2 E_{rc}}{\partial p_r \partial Q_r} = -r\beta_r f(A), \tag{21}$$

$$\frac{\partial^2 E_{rc}}{\partial \rho \partial p_r} = \frac{\partial^2 E_{rc}}{\partial p_r \partial \rho} = \alpha + r\alpha f(A). \tag{22}$$

For maximum value of E_{rc}, Eqs. (14), (16) and (18) are individually zero. Then, solving these, we have the stationary points as follows

$$Q_r^* = \left[F^{-1}\left(\frac{r - c_r}{r}\right) - \beta_r p_r^* + \alpha\rho^*\right], \tag{23}$$

$$p_r^* = \frac{\alpha\rho^* + \beta_r c_r + \mu}{2\beta_r}, \tag{24}$$

$$\rho^* = \frac{\alpha(p_r^* - c_r)}{2k}. \tag{25}$$

Now, using the optimum value of p_r^*, ρ^* in Eq. (23) and, using the optimum value of ρ^* in Eq. (24), the simplified expression of Q_r^*, p_r^*, ρ^* are given by

$$Q_r^* = \left[F^{-1}\left(\frac{r - c_r}{r}\right) + \frac{\alpha^2(\mu + c_r - \beta_r c_r) - 2k\beta_r(\mu + \beta_r c_r)}{4k\beta_r - \alpha^2}\right], \tag{26}$$

$$p_r^* = \frac{2k(\mu + \beta_r c_r) - \alpha^2 c_r}{4k\beta_r - \alpha^2}, \tag{27}$$

$$\rho^* = \frac{\alpha(\mu - \beta_r c_r)}{4k\beta_r - \alpha^2}. \tag{28}$$

Proposition 1. *The profit function E_{rc} is strictly concave function if $2k(2 + \beta_r(f(A) - r)) + r\alpha^2 f(A)(2 + 2rf(A)(1 - \beta_r)) > 0$ holds.*

Proof. The associated Hessian matrix of E_{rc} is

$$H = \begin{pmatrix} \frac{\partial^2 E_{rc}}{\partial Q_r^2} & \frac{\partial^2 E_{rc}}{\partial Q_r \partial p_r} & \frac{\partial^2 E_{rc}}{\partial Q_r \partial \rho} \\ \frac{\partial^2 E_{rc}}{\partial Q_r \partial p_r} & \frac{\partial^2 E_{rc}}{\partial p_r^2} & \frac{\partial^2 E_{rc}}{\partial p_r \partial \rho} \\ \frac{\partial^2 E_{rc}}{\partial Q_r \partial \rho} & \frac{\partial^2 E_{rc}}{\partial p_r \partial \rho} & \frac{\partial^2 E_{rc}}{\partial \rho^2} \end{pmatrix}.$$

Substituting the above second order partial derivatives, the Hessian matrix is

$$H = \begin{pmatrix} -rf(A) & -\beta_r rf(A) & r\alpha f(A) \\ -\beta_r rf(A) & -2\beta_r - \beta_r^2 rf(A) & \alpha + r\alpha f(A) \\ r\alpha f(A) & \alpha + r\alpha f(A) & -2k - r\alpha^2 f(A) \end{pmatrix}.$$

If the principal minors are alternatively negative and positive, i.e., the kth order leading principal minor D_k follows the sign of $(-1)^k$, then the profit function E_{rc} is concave, i.e., maximum at (Q_r^*, p_r^*, ρ^*). Here $D_1 = -rf(A) < 0$ as $f(A) > 0$, $D_2 = \begin{vmatrix} -rf(A) & -\beta_r rf(A) \\ -\beta_r rf(A) & -2\beta_r - \beta_r^2 rf(A) \end{vmatrix} = 2\beta_r rf(A) > 0$ and $D_3 = \begin{vmatrix} -rf(A) & -\beta_r rf(A) & r\alpha f(A) \\ -\beta_r rf(A) & -2\beta_r - \beta_r^2 rf(A) & \alpha + r\alpha f(A) \\ r\alpha f(A) & \alpha + r\alpha f(A) & -2k - r\alpha^2 f(A) \end{vmatrix} = -2k\beta_r rf(A)(2 + \beta_r (f(A) - r)) - r^2\alpha^2\beta_r f(A)^2(2 + 2rf(A)(1 - \beta_r)) < 0$ if $2k(2 + \beta_r(f(A) - r)) + r\alpha^2 f(A)(2 + 2rf(A)(1 - \beta_r)) > 0$ holds. Also, the stationary point (Q_r^*, p_r^*, ρ^*) provided in Eqs. (26)–(28) is unique. Hence, E_{rc} is strictly (i.e., unimodal) concave if $2k(2 + \beta_r(f(A) - r)) + r\alpha^2 f(A) (2 + 2rf(A)(1 - \beta_r)) > 0$ holds. The proof is completed here. □

3.1.2. Case-II: Decentralized supply chain when manufacturer is the decision maker (MDCS)

In decentralized decision making, the manufacturer and the retailer are interested to achieve maximum individual profits. Interactions between the manufacturer and the retailer are considered as a Stackelberg game. The manufacturer acts as the Stackelberg leader of the channel and the retailer is it's follower. In Stackelberg game, leader makes first move and follower then reacts by consistent playing the best move with available information. The objective of the leader is to design optimal strategies in favor of him. In this way, the manufacturer first announces the lot-size Q_r and promotional effort $\rho$of the product to the retailer. Based on the manufacturer's decision, the retailer determines the retail price p_r.

In this case, as the manufacturer determines the optimal values of Q_r and ρ. Now, differentiating E_{mr} given in Eq. (5) partially with respect to Q_r and ρ, the following derivatives are

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$$\frac{\partial E_{mr}}{\partial Q_r} = (w - c_r) - vF(A), \tag{29}$$

$$\frac{\partial^2 E_{mr}}{\partial Q_r^2} = -vf(A) < 0, \tag{30}$$

$$\frac{\partial E_{mr}}{\partial \rho} = \alpha v F(A) - 2tk\rho, \tag{31}$$

$$\frac{\partial^2 E_{mr}}{\partial \rho^2} = -\alpha^2 vf(A) - 2tk < 0, \tag{32}$$

$$\frac{\partial^2 E_{mr}}{\partial \rho \partial Q_r} = \frac{\partial^2 E_{mr}}{\partial Q_r \partial \rho} = \alpha v f(A), \tag{33}$$

$$\frac{\partial E_{rr}}{\partial p_r} = \mu - 2\beta_r p_r + \alpha\rho + \beta_r r - \beta_r (r - v)F(A), \tag{34}$$

$$\frac{\partial^2 E_{rr}}{\partial p_r^2} = -2\beta_r - \beta_r^2 (r - v)f(A) < 0. \tag{35}$$

Equating Eqs. (29) and (30) to zero and solving these, the optimal values of (Q_r, ρ) are as follows

$$Q_r^* = \left[F^{-1}\left(\frac{w - c_r}{v}\right) - \beta_r p_r^* + \alpha\rho^*\right] \tag{36}$$

and

$$\rho^* = \frac{\alpha(w - c_r)}{2tk}, \tag{37}$$

where the optimum value of p_r is obtained by solving Eq. (34) and putting above values of (Q_r^*, ρ^*) as follows:

$$p_r^* = \frac{1}{2\beta_r}\left[\mu + \frac{\alpha^2 (w - c_r)}{2tk} + r\beta_r - \frac{\beta_r (r - v)(w - c_r)}{v}\right]. \tag{38}$$

Now, substituting the values of ρ^* and p_r^* in Eq. (36), the simplified expressions of Q_r, p_r and ρ are obtained as follows:

$$Q_r^* = F^{-1}\left(\frac{w - c_r}{v}\right) - \frac{(\mu + r\beta_r)}{2} + \frac{(w - c_r)}{2}\left[\frac{\beta_r (r - v)}{v} - \frac{\alpha^2}{tk}\right], \tag{39}$$

$$p_r^* = \frac{1}{2\beta_r}\left[\mu + \frac{\alpha^2 (w - c_r)}{2tk} + r\beta_r - \frac{\beta_r (r - v)(w - c_r)}{v}\right] \tag{40}$$

and

$$\rho^* = \frac{\alpha(w - c_r)}{2tk}, \tag{41}$$

Proposition 2. *The profit functions E_{mr} and E_{rr} are strictly concave functions.*

Proof. Here, the Hessian matrix of E_{mr} is

$$H = \begin{pmatrix} \frac{\partial^2 E_{mr}}{\partial Q_r^2} & \frac{\partial^2 E_{mr}}{\partial \rho \partial Q_r} \\ \frac{\partial^2 E_{mr}}{\partial \rho \partial Q_r} & \frac{\partial^2 E_{mr}}{\partial \rho^2} \end{pmatrix}.$$

Substituting the above second order partial derivatives, the Hessian matrix is

$$H = \begin{pmatrix} -vf(A) & \alpha v f(A) \\ \alpha v f(A) & -\alpha^2 vf(A) - 2tk \end{pmatrix}.$$

The profit function E_{mr} would be concave if the value of the determinant of the Hessian matrix H is positive, i.e., $|H| > 0$ and the value of the second order partial derivatives $\frac{\partial^2 E_{mr}}{\partial Q_r^2}$ and $\frac{\partial^2 E_{mr}}{\partial \rho^2}$ are both negative.

Now, $|H| = 2tkvf(A) > 0$ as $f(A) > 0, \frac{\partial^2 E_{mr}}{\partial Q_r^2} = -vf(A) < 0$ and $\frac{\partial^2 E_{mr}}{\partial \rho^2} = -\alpha^2 vf(A) - 2tk < 0$. Moreover, the stationary point (Q_r, p_r, ρ) provided in Eqs. (39)–(41) is unique. Therefore, E_{mr} is a strictly concave function. Similarly, E_{rr} is strictly concave because $\frac{\partial^2 E_{rr}}{\partial p_r^2} = -2\beta_r - \beta_r^2 (r - v)f(A) < 0$ as $r > v$. The proof is completed here. □

3.1.3. Case-III: Decentralized supply chain when retailer is the decision maker (RDCS)

In this case, manufacturer is the follower of the decisions taken by the retailer. Then, the retailer optimize the respective lotsizes and sales price, service level and promotional effort to obtain maximum value of E_{rr}. Then, the partial derivatives of E_{rr} are

$$\frac{\partial E_{rr}}{\partial Q_r} = (r - w) - (r - v)F(A), \tag{42}$$

$$\frac{\partial^2 E_{rr}}{\partial Q_r^2} = -(r - v)f(A) < 0, \quad \forall r > v, \tag{43}$$

$$\frac{\partial E_{rr}}{\partial p_r} = (\mu - 2\beta_r p_r + \alpha\rho + \beta_r r) - (r - v)\beta_r F(A), \tag{44}$$

$$\frac{\partial^2 E_{rr}}{\partial p_r^2} = -2\beta_r - (r - v)\beta_r^2 f(A) < 0, \quad \forall r > v, \tag{45}$$

$$\frac{\partial E_{rr}}{\partial \rho} = \alpha(p_r - r) + \alpha(r - v)F(A) - 2(1 - t)k\rho, \tag{46}$$

$$\frac{\partial^2 E_{rr}}{\partial \rho^2} = -\alpha^2 (r - v)f(A) - 2(1 - t)k < 0, \quad \forall r > v, \tag{47}$$

$$\frac{\partial^2 E_{rr}}{\partial Q_r \partial \rho} = \frac{\partial^2 E_{rr}}{\partial \rho \partial Q_r} = \alpha(r - v)f(A), \tag{48}$$

$$\frac{\partial^2 E_{rr}}{\partial Q_r \partial p_r} = \frac{\partial^2 E_{rr}}{\partial p_r \partial Q_r} = (v - r)\beta_r f(A) \tag{49}$$

and

$$\frac{\partial^2 E_{rr}}{\partial \rho \partial p_r} = \frac{\partial^2 E_{rr}}{\partial p_r \partial \rho} = \alpha + \alpha(r - v)\beta_r f(A). \tag{50}$$

Equating Eqs. (42), (44) and (46) to zero and solving these, the optimal solutions are as follows:

$$Q^* = \left[F^{-1}\left(\frac{r - w}{r - v}\right) - \beta_r p_r^* + \alpha\rho^*\right], \tag{51}$$

$$p_r^* = \frac{\mu + \alpha\rho^* + \beta_r w}{2\beta_r}, \tag{52}$$

$$\rho^* = \frac{\alpha(p_r^* - w)}{2(1 - t)k}. \tag{53}$$

Now, substituting the values of ρ^* and p_r^* in the Eq. (51), the simplified expressions of (Q_r, p_r, ρ) are obtained as follows:

$$Q^* = F^{-1}\left(\frac{r - w}{r - v}\right) + \frac{\alpha^2 (\mu + \alpha r) - 2(1 - t)k\beta_r (\mu + \beta_r w)}{4\beta_r (1 - t)k - \alpha^2}, \tag{54}$$

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$$p_r^* = \frac{2(\mu + \beta_r w)(1-t)k - \alpha^2 w}{4\beta_r(1-t)k - \alpha^2} \tag{55}$$

and

$$\rho^* = \frac{\alpha(\mu - \beta_r w)}{4\beta_r(1-t)k - \alpha^2}. \tag{56}$$

Proposition 3. *The profit function* E_{rr} *is a strictly concave function if* $6k(1-t) + f(A)[2(r^2-\alpha^2)$ $+\alpha^2(r-v)(\beta_r-2)f(A) + 2(1-t)k\beta_r] > 0$.

Proof. Now, the Hessian matrix of E_{rr} is

$$H = \begin{pmatrix} \frac{\partial^2 E_{rr}}{\partial Q_r^2} & \frac{\partial^2 E_{rr}}{\partial Q_r \partial p_r} & \frac{\partial^2 E_{rr}}{\partial Q_r \partial \rho} \\ \frac{\partial^2 E_{rr}}{\partial Q_r \partial p_r} & \frac{\partial^2 E_{rr}}{\partial p_r^2} & \frac{\partial^2 E_{rr}}{\partial p_r \partial \rho} \\ \frac{\partial^2 E_{rr}}{\partial Q_r \partial \rho} & \frac{\partial^2 E_{rr}}{\partial p_r \partial \rho} & \frac{\partial^2 E_{rr}}{\partial \rho^2} \end{pmatrix}.$$

Using the values of the above second order derivatives in H, the Hessian matrix is

$$H = \begin{pmatrix} -(r-v)f(A) & \alpha(r-v)f(A) & (v-r)\beta_r f(A) \\ \alpha(r-v)f(A) & -\alpha^2(r-v)f(A) - 2(1-t)k & \alpha + \alpha(r-v)f(A) \\ (v-r)\beta_r f(A) & \alpha + \alpha(r-v)f(A) & -2\beta_r - (r-v)\beta_r^2 f(A) \end{pmatrix}.$$

If the principal minors at the stationary point are alternatively negative and positive, i.e. the kth order leading principal minor D_k takes the sign of $(-1)^k$,then the profit function E_{rr} is maximum at that stationary point. Here, $D_1 = -(r-v)f(A) < 0$ if $r > v$ and $f(A) > 0$ $D_2 = \begin{vmatrix} -(r-v)f(A) & \alpha(r-v)f(A) \\ \alpha(r-v)f(A) & -\alpha^2(r-v)f(A) - 2(1-t)k \end{vmatrix} = 2(1-t)k(r-v)$ $f(A) > 0$ as $r > v$ and $t < 1$ $D_3 = |-(r-v)f(A)\ \ \alpha(r-v)$ $f(A)(v-r)\beta_r f(A)\alpha(r-v)f(A)-\ \alpha^2(r-v)f(A) - 2(1-t)k\alpha\ +\alpha(r-v)$ $f(A)(v-r)\beta_r f(A)\alpha\ +\alpha(r-v)f(A) - 2\beta_r - (r-v)\beta_r^2 f(A)| = -6k(1-t)$ $(r-v)^2\beta_r f(A) - (r-v)^2\beta_r f^2(A)[2(r^2-\alpha^2) + \alpha^2(r-v)(\beta_r-2)\ \ f(A) +$ $2(1-t)k\beta_r] < 0$ if $6k(1-t) + f(A)[2(r^2-\alpha^2) + \alpha^2(r-v)(\beta_r-2)$ $f(A) + 2(1-t)k\beta_r] > 0$ holds. The stationary point provided in Eqs. (54)–(56) are unique. Therefore, the profit function E_{rr} is strictly concave if $6k(1-t) + f(A)[2(r^2-\alpha^2) + \alpha^2(r-v)(\beta_r-2)f(A) +$ $2(1-t)k\beta_r] > 0$ holds. The proof is completed here. □

3.2. Direct online e-tail channel environment (EC)

When the manufacturer does business through direct online e-tail channel (EC) only, there is no retailer between the manufacturer and the customers. The order of the product is directly placed through the internet and the product is directly shipped from the warehouse of the manufacturer to the address specified by the end customer. The expected profit $E_{ec}(Q_e, p_e, s)$ of the manufacturer in the e-tail channel is provided in Eq. (8). In this case, the decision variables of the manufacturer are the lot size Q_e, the e-tail price p_e and service level assurance s.

The partial derivatives of $E_{ec}(Q_e, p_e, s)$ with respect to Q_e, p_e and s are

$$\frac{\partial E_{ec}}{\partial Q_e} = (r_e - c_e) - r_e F(B), \tag{56}$$

$$\frac{\partial^2 E_{ec}}{\partial Q_e^2} = -r_e f(B) < 0, \tag{57}$$

$$\frac{\partial E_{ec}}{\partial p_e} = (\mu - 2\beta_e p_e + \delta s + r_e\beta_e) - r_e\beta_e F(B), \tag{58}$$

$$\frac{\partial^2 E_{ec}}{\partial p_e^2} = -2\beta_e - r_e\beta_e^2 f(B) < 0, \tag{59}$$

$$\frac{\partial E_{ec}}{\partial s} = \delta p_e - r_e\delta + r_e\delta F(B) - ns, \tag{60}$$

$$\frac{\partial^2 E_{ec}}{\partial s^2} = -r_e\delta^2 f(B) - n < 0, \tag{61}$$

$$\frac{\partial^2 E_{ec}}{\partial Q_e \partial p_e} = \frac{\partial^2 E_{ec}}{\partial p_e \partial Q_e} = -r_e\beta_e f(B), \tag{62}$$

$$\frac{\partial^2 E_{ec}}{\partial Q_e \partial s} = \frac{\partial^2 E_{ec}}{\partial s \partial Q_e} = r_e\delta f(B) \tag{63}$$

and

$$\frac{\partial^2 E_{ec}}{\partial s \partial p_e} = \frac{\partial^2 E_{ec}}{\partial p_e \partial s} = \delta + r_r\delta\beta_e f(B). \tag{64}$$

Equating Eqs. (56), (58) and (60) to zero and solving these, the optimal values of the decision variables are as follows:

$$Q_e^* = F^{-1}(\frac{r_e - c_e}{r_e}) - \beta_e p_e^* + \delta s^*, \tag{65}$$

$$p_e^* = \frac{\mu + \delta s^* + \beta_e c_e}{2\beta_e} \tag{66}$$

and

$$s^* = \frac{\delta(p_e^* - c_e)}{n}. \tag{67}$$

Substituting the values of s^* and p_e^* in the Eq. (65), the explicit optimal values of Q_e, p_e and s are as follows:

$$Q_e^* = F^{-1}\left(\frac{r_e - c_e}{r_e}\right) + \frac{\delta^2\mu - n\beta_e\mu - n\beta_e^2 c_e}{2n\beta_e - \delta^2}, \tag{68}$$

$$p_e^* = \frac{n\mu + n\beta_e c_e - \delta^2 c_e}{2n\beta_e - \delta^2} \tag{69}$$

and

$$s^* = \frac{\delta(\mu - \beta_e c_e)}{2n\beta_e - \delta^2}. \tag{70}$$

Proposition 4. *The profit function* E_{ec} *is a strictly concave function if* $\{2 + r_e\beta_e f(B)\}\{n$ $+r_e\delta^2 f(B)\} > r_e f(B)(n\beta_e + \delta) + \delta^2\{(1 + r_r\beta_e f(B))^2 - r_e f(B)\}$*holds.*

Proof. The Hessian matrix of $E_{ec}(Q_e, p_e, \rho)$ is

$$H = \begin{pmatrix} \frac{\partial^2 E_{ec}}{\partial Q_e^2} & \frac{\partial^2 E_{ec}}{\partial Q_e \partial p_e} & \frac{\partial^2 E_{ec}}{\partial Q_e \partial s} \\ \frac{\partial^2 E_{ec}}{\partial Q_e \partial p_e} & \frac{\partial^2 E_{ec}}{\partial p_e^2} & \frac{\partial^2 E_{ec}}{\partial p_e \partial s} \\ \frac{\partial^2 E_{ec}}{\partial Q_e \partial s} & \frac{\partial^2 E_{ec}}{\partial p_e \partial s} & \frac{\partial^2 E_{ec}}{\partial s^2} \end{pmatrix}.$$

At the values of respective second order partial derivatives, the Hessian matrix is

$$H = \begin{pmatrix} -r_e f(B) & -r_e\beta_e f(B) & r_e\delta f(B) \\ -r_e\beta_e f(B) & -2\beta_e - \beta_e^2 r_e f(B) & \delta + r_r\delta\beta_e f(B) \\ r_e\delta f(B) & \delta + r_r\delta\beta_e f(B) & -r_e\delta^2 f(B) - n \end{pmatrix}.$$

If the principal minors at the stationary point (Q_e, p_e, ρ) are alternatively negative and positive, i.e. the kth order leading principal

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minor D_k takes the sign of $(-1)^k$,then the profit function E_{ec} is maximum and it is unimodal as the stationary point is unique.

Here, $D_1 = -r_e f(B) < 0$ as $f(B) > 0$, $D_2 = \begin{vmatrix} -r_e f(B) & -r_e\beta_e f(B) \\ -r_e\beta_e f(B) & -2\beta_e - \beta_e^2 r_e f(B) \end{vmatrix} = 2r_e\beta_e f(B) > 0$ and $D_3 = \begin{vmatrix} -r_e f(B) & -r_e\beta_e f(B) & r_e\delta f(B) \\ -r_e\beta_e f(B) & -2\beta_e - \beta_e^2 r_e f(B) & \delta + r_r\delta\beta_e f(B) \\ r_e\delta f(B) & \delta + r_r\delta\beta_e f(B) & -r_e\delta^2 f(B) - n \end{vmatrix} = -r_e\beta_e f(B)$ $[\{2 + r_e\beta_e f(B)\}\{n + r_e\delta^2 f(B)\} - r_e f(B)(n\beta_e + \delta) \quad -\delta^2\{(1 + r_r\beta_e f(B))^2$ $-r_e f(B)\}] < 0$ if $\{2 + r_e\beta_e f(B)\}\{n + r_e\delta^2 f(B)\} > r_e f(B)(n\beta_e + \delta) \; +\delta^2$ $\{(1 + r_r\beta_e f(B))^2 - r_e f(B)\}$ holds.

Therefore, the profit function E_{ec} is strictly concave function if $\{2 + r_e\beta_e f(B)\}\{n + r_e\delta^2 f(B)\} > r_e f(B)(n\beta_e + \delta) \quad +\delta^2\{(1 + r_r\beta_e f(B))^2$ $-r_e f(B)\}$ holds. The proof is completed here. □

3.3. Dual channel environment (DC)

In this section, a centralized dual-channel supply channel is considered in which the manufacturer and the retailer are vertically integrated with traditional channel as well as direct online e-tail channel. The integrated expected profit function EIP_{dc} given in Eq. (13) contains the decision variables Q_r, Q_e, p_r, p_e, ρ and s.

The partial derivatives of $EIP_{dc}(Q_r, Q_e, p_r, p_e, \rho, s)$ with respect to the decision variables Q_r, Q_e, p_r, p_e, ρ and s are

$$\frac{\partial EIP_{dc}}{\partial Q_r} = -c_r + r - rF(D), \tag{71}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r^2} = -rf(D) < 0, \tag{72}$$

$$\frac{\partial EIP_{dc}}{\partial Q_e} = -c_e + r_e - r_e F(G), \tag{73}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_e^2} = -r_e f(G) < 0, \tag{74}$$

$$\begin{aligned}\frac{\partial EIP_{dc}}{\partial p_r} = {} & (\mu - 2\beta_r p_r + \gamma_e p_e + \alpha\rho + \gamma_r p_e + r\beta_r - r_e\gamma_r) \\ & - r\beta_r F(D) + \gamma_r r_e F(G),\end{aligned} \tag{75}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_r^2} = -2\beta_r - r\beta_r^2 f(D) - r_e\gamma_r^2 f(G) < 0, \tag{76}$$

$$\begin{aligned}\frac{\partial EIP_{dc}}{\partial p_e} = {} & (\mu - 2\beta_e p_e + \gamma_r p_r + \delta s + \gamma_e p_r - r\gamma_e - r_e\beta_e) \\ & + r\gamma_e F(D) - r_e\beta_e F(G),\end{aligned} \tag{77}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_e^2} = -2\beta_e - r\gamma_e^2 f(D) - r_e\beta_e^2 f(G) < 0, \tag{78}$$

$$\frac{\partial EIP_{dc}}{\partial s} = \delta p_e - \delta r_e + \delta r_e F(G) - ns, \tag{79}$$

$$\frac{\partial^2 EIP_{dc}}{\partial s^2} = -r_e\delta^2 f(G) - n < 0, \tag{80}$$

$$\frac{\partial EIP_{dc}}{\partial \rho} = \alpha p_r - \alpha r + \alpha r F(D) - 2k\rho, \tag{81}$$

$$\frac{\partial^2 EIP_{dc}}{\partial \rho^2} = -r\alpha^2 f(D) - 2k < 0, \tag{82}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r \partial Q_e} = \frac{\partial^2 EIP_{dc}}{\partial Q_e \partial Q_r} = 0, \tag{83}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r \partial p_r} = \frac{\partial^2 EIP_{dc}}{\partial p_r \partial Q_r} = -r\beta_r f(D), \tag{84}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r \partial p_e} = \frac{\partial^2 EIP_{dc}}{\partial p_e \partial Q_r} = r\gamma_e f(D), \tag{85}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r \partial \rho} = \frac{\partial^2 EIP_{dc}}{\partial \rho \partial Q_r} = r\alpha f(D), \tag{86}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_r \partial s} = \frac{\partial^2 EIP_{dc}}{\partial s \partial Q_r} = 0, \tag{87}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_e \partial p_r} = \frac{\partial^2 EIP_{dc}}{\partial p_r \partial Q_e} = r_e\gamma_r f(G), \tag{88}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_e \partial p_e} = \frac{\partial^2 EIP_{dc}}{\partial p_e \partial Q_e} = -r_e\beta_e f(G), \tag{89}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_e \partial \rho} = \frac{\partial^2 EIP_{dc}}{\partial \rho \partial Q_e} = 0, \tag{90}$$

$$\frac{\partial^2 EIP_{dc}}{\partial Q_e \partial s} = \frac{\partial^2 EIP_{dc}}{\partial s \partial Q_e} = r_e\delta f(G), \tag{91}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_r \partial p_e} = \frac{\partial^2 EIP_{dc}}{\partial p_e \partial p_r} = (\gamma_e + \gamma_r) + r\gamma_e\beta_r f(D) - r_e\beta_e\gamma_r f(G), \tag{92}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_r \partial \rho} = \frac{\partial^2 EIP_{dc}}{\partial \rho \partial p_r} = \alpha(1 + r\beta_r f(D)), \tag{93}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_r \partial s} = \frac{\partial^2 EIP_{dc}}{\partial s \partial p_r} = -r_e\gamma_r\delta f(G), \tag{94}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_e \partial \rho} = \frac{\partial^2 EIP_{dc}}{\partial \rho \partial p_e} = -\alpha r\gamma_e f(D), \tag{95}$$

$$\frac{\partial^2 EIP_{dc}}{\partial p_e \partial s} = \frac{\partial^2 EIP_{dc}}{\partial s \partial p_e} = \delta(1 + r_e\beta_e f(G)), \tag{96}$$

$$\frac{\partial^2 EIP_{dc}}{\partial \rho \partial s} = \frac{\partial^2 EIP_{dc}}{\partial s \partial \rho} = 0. \tag{97}$$

Equating Eqs. (71), (73), (75), (77), (79) and (81) to zero and solving these, the required optimal solutions of the decision variables are as follows:

$$Q_r^* = F^{-1}(\frac{r - c_r}{r}) - \beta_r p_r^* + \gamma_e p_e^* + \alpha\rho^*, \tag{98}$$

$$Q_e^* = F^{-1}(\frac{r_e - c_e}{r_e}) - \beta_e p_e^* + \gamma_r p_r^* + \delta s^*, \tag{99}$$

where $p_r^*, p_e^*, \rho^*, s^*$ are given by

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$$p_r^* = \frac{\alpha^2 c_r(2n\beta_e - \delta^2) + 2k((\delta^2 c_e - n(\mu + c_e\beta_e - c_r\gamma_e))(\gamma_e + \gamma_r)) - (2n\beta_e - \delta^2)(\mu + c_r\beta_r - c_e\gamma_r)}{2n\alpha^2\beta_e - \alpha^2\delta^2 + 4k\delta^2\beta_r - 2kn(4\beta_e\beta_r - (\gamma_e + \gamma_r)^2)}, \tag{100}$$

$$p_e^* = \frac{(n\mu + n\beta_e c_e - \delta^2 c_e)(\alpha^2 - 4k\beta_r) + n\alpha^2 c_r\gamma_r + 2kn(\gamma_e + \gamma_r)(c_r\beta_r - \mu - c_e\gamma_r)}{2n\alpha^2\beta_e - \alpha^2\delta^2 + 4k\delta^2 + 2kn((\gamma_e + \gamma_r) - 4\beta_e\beta_r)}, \tag{101}$$

$$\rho^* = \frac{\frac{\alpha c_r}{2k} - (\alpha^3\delta^2 c_r - 2n\alpha^3 c_r\beta_e) - 2k\alpha((\delta c_e - n\mu + nc_e\beta_e + nc_r\gamma_e)(\gamma_e + \gamma_r) - (2n\beta_e - \delta^2)(\mu + c_r\beta_r - c_e\gamma_r))}{2k(2n\alpha^2\beta_e - \alpha^2\delta^2 + 4k\delta^2\beta_r) + 2kn((\gamma_e + \gamma_r)^2 - 4\beta_e\beta_r)} \tag{102}$$

and

$$s^* = \frac{\alpha^2\delta(\mu - \beta_e c_e + c_r\gamma_r) + 4k\delta\beta_r(c_e\beta_e - \mu) + 2k\delta(\gamma_e + \gamma_r)(c_r\beta_r - c_e\gamma_e - \mu)}{2n\alpha^2\beta_e - \alpha^2\delta^2 + 4k\delta^2\beta_r + 2kn(4\beta_e\beta_r - (\gamma_e + \gamma_r)^2)}. \tag{103}$$

Proposition 5. *The profit function EIP_{dc} is a strictly concave function, i.e., EIP_{dc} attains its maximum value at a unique stationary point $(Q_r^*, Q_e^*, p_r^*, p_e^*, \rho^*, s^*)$ if $4\beta_e\beta_r - (\gamma_r - r\beta_r\gamma_e f(D))^2 + r_e(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) + r_e(2\beta_e\gamma_r f(G) - 1 - r\beta_r f(D))) > 0,$ $k(\gamma_r - r\beta_r\gamma_e f(D))^2 - \beta_e(\alpha^2 - 4k\beta_r) - r_e k(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) - (1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))) > 0$ and $(\alpha^2 - 4k\beta_r)(\delta^2 - 2n\beta_e) - 2kn\gamma_r^2 + 2knr\beta_r\gamma_e f(D)(2\gamma_r - r\beta_r\gamma_e f(D)) + 4knr_e(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(r\beta_r\gamma_e f(D) - r) > 0$ hold simultaneously.*

Proof. The Hessian matrix of EIP_{dc} is

$$H = \begin{pmatrix}
\frac{\partial^2 EIP_{dc}}{\partial Q_r^2} & \frac{\partial^2 EIP_{dc}}{\partial Q_r \partial Q_e} & \frac{\partial^2 EIP_{dc}}{\partial Q_r \partial p_r} & \frac{\partial^2 EIP_{dc}}{\partial Q_r \partial p_e} & \frac{\partial^2 EIP_{dc}}{\partial Q_r \partial \rho} & \frac{\partial^2 EIP_{dc}}{\partial Q_r \partial s} \\
\frac{\partial^2 EIP_{dc}}{\partial Q_e \partial Q_r} & \frac{\partial^2 EIP_{dc}}{\partial Q_e^2} & \frac{\partial^2 EIP_{dc}}{\partial Q_e \partial p_r} & \frac{\partial^2 EIP_{dc}}{\partial Q_e \partial p_e} & \frac{\partial^2 EIP_{dc}}{\partial Q_e \partial \rho} & \frac{\partial^2 EIP_{dc}}{\partial Q_e \partial s} \\
\frac{\partial^2 EIP_{dc}}{\partial p_r \partial Q_r} & \frac{\partial^2 EIP_{dc}}{\partial p_r \partial Q_e} & \frac{\partial^2 EIP_{dc}}{\partial p_r^2} & \frac{\partial^2 EIP_{dc}}{\partial p_r \partial p_e} & \frac{\partial^2 EIP_{dc}}{\partial p_r \partial \rho} & \frac{\partial^2 EIP_{dc}}{\partial p_r \partial s} \\
\frac{\partial^2 EIP_{dc}}{\partial p_e \partial Q_r} & \frac{\partial^2 EIP_{dc}}{\partial p_e \partial Q_e} & \frac{\partial^2 EIP_{dc}}{\partial p_e \partial p_r} & \frac{\partial^2 EIP_{dc}}{\partial p_e^2} & \frac{\partial^2 EIP_{dc}}{\partial p_e \partial \rho} & \frac{\partial^2 EIP_{dc}}{\partial p_e \partial s} \\
\frac{\partial^2 EIP_{dc}}{\partial \rho \partial Q_r} & \frac{\partial^2 EIP_{dc}}{\partial \rho \partial Q_e} & \frac{\partial^2 EIP_{dc}}{\partial \rho \partial p_r} & \frac{\partial^2 EIP_{dc}}{\partial \rho \partial p_e} & \frac{\partial^2 EIP_{dc}}{\partial \rho^2} & \frac{\partial^2 EIP_{dc}}{\partial \rho \partial s} \\
\frac{\partial^2 EIP_{dc}}{\partial s \partial Q_r} & \frac{\partial^2 EIP_{dc}}{\partial s \partial Q_e} & \frac{\partial^2 EIP_{dc}}{\partial s \partial p_r} & \frac{\partial^2 EIP_{dc}}{\partial s \partial p_e} & \frac{\partial^2 EIP_{dc}}{\partial s \partial \rho} & \frac{\partial^2 EIP_{dc}}{\partial s^2}
\end{pmatrix}.$$

Putting the values of the second order derivatives into the above, the Hessian matrix is

$$H = \begin{pmatrix}
-rf(D) & 0 & -r\beta_r f(D) & r\gamma_e f(D) & r\alpha f(D) & 0 \\
0 & -r_e f(G) & r_e\gamma_r f(G) & -r_e\beta_e f(G) & 0 & r_e\delta f(G) \\
-r\beta_r f(D) & r_e\gamma_r f(G) & h_1 & h_2 & \alpha(1 + r\beta_r f(D)) & -r_e\gamma_r\delta f(G) \\
r\gamma_e f(D) & -r_e\beta_e f(G) & h_2 & h_3 & -\alpha r\gamma_e f(D) & \delta(1 + r_e\beta_e f(G)) \\
r\alpha f(D) & 0 & \alpha(1 + r\beta_r f(D)) & -\alpha r\gamma_e f(D) & -r\alpha^2 f(D) - 2k & 0 \\
0 & r_e\delta f(G) & -r_e\gamma_r\delta f(G) & \delta(1 + r_e\beta_e f(G)) & 0 & -r_e\delta^2 f(G) - n
\end{pmatrix},$$

where $h_1 = -2\beta_r - r\beta_r^2 f(D) - r_e\gamma_r^2 f(G)$ and $h_2 = (\gamma_e + \gamma_r) + r\gamma_e\beta_r f(D) - r_e\beta_e\gamma_r f(G)$ and $h_3 = -2\beta_e - r\gamma_e^2 f(D) - r_e\beta_e^2 f(G)$. Now, $D_1 = -rf(D) < 0$ as $f(D) > 0,$ $D_2 = \begin{vmatrix} -rf(D) & 0 \\ 0 & -r_e f(G) \end{vmatrix} = rr_e f(D)f(G) > 0$ as $f(D)$ and $f(G) > 0,$ $D_3 = \begin{vmatrix} -rf(D) & 0 & -r\beta_r f(D) \\ 0 & -r_e f(G) & r_e\gamma_r f(G) \\ -r\beta_r f(D) & r_e\gamma_r f(G) & h_1 \end{vmatrix} = -2rr_e\beta_r f(D)f(G) < 0$ as $f(D) > 0$ and $f(G) > 0,$ $D_4 = \begin{vmatrix} -rf(D) & 0 & -r\beta_r f(D) & r\gamma_e f(D) \\ 0 & -r_e f(G) & r_e\gamma_r f(G) & -r_e\beta_e f(G) \\ -r\beta_r f(D) & r_e\gamma_r f(G) & h_1 & h_2 \\ r\gamma_e f(D) & -r_e\beta_e f(G) & h_2 & h_3 \end{vmatrix} = rr_e\ f(D)f(G)$ $[4\beta_e\beta_r - (\gamma_r - r\beta_r\gamma_e f(D))^2 - r_e^2(1 + r\beta_r\ f(D) - 2\beta_e\gamma_r f(G))^2 + 2r_e(r\beta_r\gamma_e f(D) - \gamma_2)(1 + r\beta_r f(D)\ -2\beta_e\gamma_r f(G))^2] = rr_e f(D)f(G)[4\beta_e\beta_r - (\gamma_r - r\beta_r\gamma_e f(D))^2 + r_e(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) + r_e(2\beta_e\gamma_r f(G) - 1 - r\beta_r f(D)))] > 0$ if $4\beta_e\beta_r - (\gamma_r - r\beta_r\gamma_e f(D))^2 + r_e(1 + r\beta_r f(D)\ -2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) + r_e(2\beta_e\gamma_r f(G) - 1 - r\beta_r f(D))) > 0$ holds, $D_5 = \begin{vmatrix} -rf(D) & 0 & -r\beta_r f(D) & r\gamma_e f(D) & r\alpha f(D) \\ 0 & -r_e f(G) & r_e\gamma_r f(G) & -r_e\beta_e f(G) & 0 \\ -r\beta_r f(D) & r_e\gamma_r f(G) & h_1 & h_2 & \alpha(1 + r\beta_r f(D)) \\ r\gamma_e f(D) & -r_e\beta_e f(G) & h_2 & h_3 & -\alpha r\gamma_e f(D) \\ r\alpha f(D) & 0 & \alpha(1 + r\beta_r f(D)) & -\alpha r\gamma_e f(D) & -r\alpha^2 f(D) - 2k \end{vmatrix} =$ $2rr_e f(D)f(G)[\beta_e(4k\beta_r - \alpha^2) + k(\gamma_r - r\beta_r\gamma_e f(D))^2 - 2r_e k\ (r\beta_r\gamma_e f(D) - \gamma_r)(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G)) + kr_e^2(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))^2] = -2rr_e f(D)f(G)[k(\gamma_r - r\beta_r\gamma_e f(D))^2 - \beta_e(\alpha^2 - 4k\beta_r) - r_e k(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) - (1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G)))] < 0$ if $k(\gamma_r - r\beta_r\gamma_e f(D))^2 - \beta_e(\alpha^2 - 4k\beta_r) - r_e k(1 + r\beta_r f(D) - 2\beta_e\gamma_r\ f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) - (1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))) > 0$ holds and $D_6 = \begin{vmatrix} -rf(D) & 0 & -r\beta_r f(D) & r\gamma_e f(D) & r\alpha f(D) & 0 \\ 0 & -r_e f(G) & r_e\gamma_r f(G) & -r_e\beta_e f(G) & 0 & r_e\delta f(G) \\ -r\beta_r f(D) & r_e\gamma_r f(G) & h_1 & h_2 & -r_e\gamma_r\delta f(G) & \\ r\gamma_e f(D) & -r_e\beta_e f(G) & h_2 & h_3 & -\alpha r\gamma_e f(D) & \delta(1 + r_e\beta_e f(G)) \\ r\alpha f(D) & 0 & \alpha(1 + r\beta_r f(D)) & -\alpha r\gamma_e f(D) & -r\alpha^2 f(D) - 2k & 0 \\ 0 & r_e\delta f(G) & -r_e\gamma_r\delta f(G) & \delta(1 + r_e\beta_e f(G)) & 0 & -r_e\delta^2 f(G) - n \end{vmatrix} =$ $rr_e f(D)f(G)[(\alpha^2 - 4k\beta_r)(\delta^2 - 2n\beta_e) - 2kn\gamma_r^2 + 2knr\beta_r\gamma_e f(D)(2\gamma_r - r\beta_r\gamma_e f(D)) + 4knr_e\ (1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(r\beta_r\gamma_e f(D) - r)] > 0$ if $[(\alpha^2 - 4k\beta_r)(\delta^2 - 2n\beta_e) - 2kn\gamma_r^2 + 2knr\beta_r\gamma_e f(D)(2\gamma_r - r\beta_r\gamma_e f(D)) + 4knr_e(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(r\beta_r\gamma_e f(D) - r)] > 0$ holds. Here, the stationary point provided in Eqs. (98)–(103) is unique. Consequently, the principal minors of the Hessian matrix are alternatively negative and positive when $4\beta_e\beta_r - (\gamma_r - r\beta_r\gamma_e f(D))^2 + r_e(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) + r_e(2\beta_e\gamma_r f(G) - 1 - r\beta_r f(D))) > 0,$ $k(\gamma_r - r\beta_r\gamma_e f(D))^2 - \beta_e(\alpha^2 - 4k\beta_r) - r_e k(1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))(2(r\beta_r\gamma_e f(D) - \gamma_r) - (1 + r\beta_r f(D) - 2\beta_e\gamma_r f(G))) > 0$ and $(\alpha^2 - 4k\beta_r)(\delta^2 - 2n\beta_e) - 2kn\gamma_r^2 + 2knr\beta_r\gamma_e f(D)(2\gamma_r - r\beta_r\gamma_e f(D)) + 4knr_e$

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Table 1
Optimal solution in traditional retail channel environment.

Scenarios under RC	Optimal values of variables					
	Q_r^*	p_r^* ($)	ρ^*	E_{rc}^* ($)	E_{rr}^* ($)	E_{mr}^* ($)
Case-I(CS)	87.523	70.60	4.0618	5618.38	–	–
Case-II (MDCS)	85.7208	71.03	0.03	–	5337.76	252.729
Case-III (RDCS)	86.2236	71.9908	9.39735	–	5183.29	205.693

Table 2
Optimal solution in direct online channel environment.

Scenarios under EC	Optimal values of variables			
	Q_e^*	p_e^* ($)	s^*	E_{ec}^* ($)
Case-IV	113.404	55.56	10.5401	5259.86

$(1+r\beta_r f(D)-2\beta_e\gamma_r f(G))(r\beta_r\gamma_e f(D)-r)>0$ hold simultaneously. Hence, EIP_{dc} is strictly concave function. The proof is completed here. □

4. Numerical example

The values of the key parameters in appropriate units are considered as follows: $\{f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{x-\mu_i}{\sigma_i})^2}, \forall-\infty\leqslant x\leqslant\infty\}, \mu=150$ units, $\sigma=2$ units, $\alpha=0.4, \beta_r=0.9, \beta_e=0.7, \gamma_r=0.3, \gamma_e=0.2, c_r=\$6, c_e=\$8, w=\$9, r=\$10, r_e=\$12, v=\$7, k=4, t=0.15, n=3$. Then, the optimal solutions for different cases are shown in the following Tables 1–8. Table 1 shows the optimal solution for the retail-channel environment (RC). In Table 1, it is observed that integrated system is best strategy for the members of the chain and the required profit sharing and their respective profits are given in Table 1. In the previous analysis, the profit functions are concave and the required optimal solutions are provided in this Table. For Case-I(CS), the hessian matrix at the respective optimal solutions is negative definite as the eigenvalues are $(-2.06104, -0.662143, -0.373959)$ all negative. In Case-II (MDCS), the hessian matrix at the respective optimal solutions is negative definite, as the eigenvalues are $(-2.63336, -0.287042)$ negative. In Case-III(RDCS), the hessian matrix at the respective optimal solutions is negative definite, because the eigenvalues are $(-3.74002, -1.56679, -0.648655)$. In Case-IV(EC), the hessian matrix at the respective optimal solutions is negative definite due to negative eigenvalues $(-5.34022, -1.72624, -0.643392)$.

Based on optimal values provided in Tables 1–8, it is observed that dual channel equipped with promotional effort and service level assurance generates more profit than the other alternatives. Moreover, profit through dual channel with promotional effort and without service level assurance is lower than the profit in dual channel equipped with both the efforts. Similarly, profit in DC without promotional effort and with service level assurance is higher than the expected profit when service level assurance is not applied. Therefore, DC is more effective if sales service level assurance is offered to the customers at free of cost. The above numerical study suggests to the management of the chain to incorporate service level assurance in e-tail channel and promotional effort in retail channel to attract the customers to buy more. Using promotional effort and service level assurance, management of the chain obtains a better solution to help the company to communicate the value of the serviced they deliver and identify service options that will help in advance their businesses further.

Table 4
Optimal solution in traditional retail channel environment taking there is no promotional effort, i.e., $\rho=0$.

Scenarios under RC	Optimal values of variables				
	Q_r^*	p_r^* ($)	E_{rc}^* ($)	E_{rr}^* ($)	E_{mr}^* ($)
Case-I(CS)	86.5563	69.93	5558.21	–	–
Case-II(MDCS)	70.7185	87.69	–	5590.95	207.74
Case-III(RDCS)	85.108	71.15	–	5340.39	–

Table 5
Optimal solution in direct online channel environment without service level assurance, i.e., $s=0$.

Scenarios under EC	Optimal values of variables		
	Q_e^*	p_e^* ($)	E_{ec}^* ($)
Case-IV	111.017	54.46	5189.13

Table 6
Optimal solution in dual channel environment without the effect of promotional effort and service level assurance, i.e., $\rho=0$ and $s=0$.

Scenarios under DC	Optimal values of variables				
	Q_r^*	Q_e^*	p_r^* ($)	p_e^* ($)	EIP_{dc}^* ($)
Case-V	57.5758	93.3204	411.636	358.618	19762.20

Table 7
Optimal solution in dual channel environment taking the promotional effort non-zero but sales service effort zero, i.e., $\rho\neq0$ and $s=0$.

Scenarios under DC	Optimal values of variables					
	Q_r^*	Q_e^*	p_r^* ($)	p_e^* ($)	ρ	EIP_{dc}^* ($)
Case-V	66.3984	78.4335	129.89	156.67	6.19492	19914.20

Table 8
Optimal solution in dual channel environment taking both the promotional effort zero but sales service effort non-zero, i.e., $\rho=0$ and $s\neq0$.

Scenarios under DC	Optimal values of variables					
	Q_r^*	Q_e^*	p_r^* ($)	p_e^* ($)	s	EIP_{dc}^* ($)
Case-V	65.0062	80.6833	129.37	159.74	15.1742	20099.70

5. Conclusion

In the present article, a management problem related to supply chain management consisting of one manufacturer and one retailer is studied to find out optimal order quantities, sales prices, promotional effort and service level, assuming uncertainty in the market

Table 3
Optimal solution in dual channel environment.

Scenarios under DC	Optimal values of variables						
	Q_r^*	Q_e^*	p_r^* ($)	p_e^* ($)	ρ^*	s^*	EIP_{dc}^* ($)
Case-V	66.2272	80.769	130.92	160.30	6.24586	15.2306	20253.80

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demand.In addition when there is uncertainty in the demand of the traditional channel(brick-and-mortar retail store), the manufacturer will design a special strategy to cope with different channel setting and to obtain more expected profit. The traditional retail channel is owned by the other member of the channel that is the retailer and the other is an e-channel in which customers place orders through the Internet. Furthermore, a decentralized system cannot always outperform a system with a integrated optimistic and pessimistic market setting well. Finally, for a decentralized system, the more optimistic market setting causes higher optimal selling prices otherwise, the more pessimistic the market setting causes lower optimal selling prices. Consequently, the integrated system suggests the optimal selling prices for the channel members to avoid pessimistic and optimistic situations so that the members may achieve optimum profits according to their contribution to the coordination of the channel. The solution of service level agreement can help the organization to better articulate the value of information technology in business terms and understand service performance from the top to down of the chain. Management often uses promotional effort and service level assurance in practice to coordinate supply chain, and it is observed that these mechanisms are directionally effective.

The proposed model has some limitations such as sales prices, promotional efforts and service level assurance are deterministic and continuous variables. The deterministic limitations can be waived considering uncertain pricing, promotional effort and service level and the continuous feature can be relaxed by considering discrete decision variables in future. This model might be extended immediately taking into account of trade credit financing strategy and supply disruption, i.e., lead time of delivery of the products.

Uncited reference

Zhanga, Lianga, Yua, and Yu (2007).

References

Balasubramanian, S. (1998). Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers. *Marketing Science, 17*, 181–195.

Cai, G. (2010). Channel selection and coordination in dual-channel supply chains. *Journal of Retailing, 86*, 22–36.

Cai, G. G., Zhang, Z., & Zhang, M. (2009). Game theoretical perspectives on dual-channel supply chain competition with price discounts and pricing schemes. *International Journal of Production Economics, 117*, 80–96.

Cattani, K., Gilland, W., & Swaminathan, J. M. (2004). Coordinating Internet and traditional supply chains. In Simchi-Levi David, Wu David, & Shen Max (Eds.), *Handbook of quantitative supply chain analysis: Modeling in the e-business era*. Elsevier Publishers.

Chen, K. Y., Kaya, M., & Ozer, O. (2008). Dual sales channel management with service competition. *Manufacturing and Service Operations Management, 10*, 654–675.

Chiang, W. K., Chhajed, D., & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply chain design. *Management Science, 49*, 1–20.

Choi, T. M., Li, Y., & Xu, L. (2013). Channel leadership, performance and coordination in closed loop supply chains. *International Journal of Production Economics, 146*, 371–380.

Choi, S. C. (1996). Pricing competition in a duopoly common retailer channel. *Journal of Retailing, 72*, 117–134.

Choi, S., & Fredj, K. (2013). Price competition and store competition: Store brand vs. national brand. *European Journal of Operational Research, 225*, 166–178.

Cao, E. (2014). Coordination of dual-channel supply chains under demand disruptions management decisions. *International Journal of Production Research, 52*, 7114–7131.

Cardenas-Barron, L. E., Taleizadeh, A. A., & Trevino-Garza, G. (2012). An improved solution to replenishment lot size problem with discontinuous issuing policy and rework, and the multi-delivery policy into economic production lot size problem with partial rework. *Expert Systems with Applications, 39*, 13540–13546.

Chung, K. J., Cardenas-Barron, L. E., & Ting, P. S. (2014). An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade credit. *International Journal of Production Economics, 155*, 310–317.

Dewan, R., Jing, B., & Seidmann, A. (2003). Product customization and price competition on the internet. *Management Science, 49*, 1055–1070.

Edirisinghe, N. C. P., Bichescu, B., & Shi, X. (2011). Equilibrium analysis of supply chain structures under power imbalance. *European Journal of Operational Research, 214*, 568–578.

Garcia-Laguna, J., San-José, L. A., Cardenas-Barron, L. E., & Sicilia, J. (2010). The integrality of the lot size in the basic EOQ and EPQ models: Applications to other production-inventory models. *Applied Mathematics and Computation, 216*, 1660–1672.

Gerchak, Y., & Wang, Y. (2004). Revenue-sharing vs. wholesale-price contracts in assembly systems with random demand. *Production and Operations Management, 13*, 23–33.

Huang, S., Yang, C., & Liu, H. (2013). Pricing and production decisions in a dual channel supply chain when production costs are disrupted. *Economic Modelling, 30*, 521–538.

Hotelling, H. (1929). Stability in competition. *Economic Journal, 39*, 41–57.

Lal, R., & Sarvary, M. (1999). When and how is the Internet likely to decrease price competition? *Marketing Science, 18*, 485–503.

Lippmand, S., & Macardle, K. (1997). The competitive newsboy. *Operations Research, 45*, 54–65.

Lu, J.-C., Tsao, Y.-C., & Charoensiriwath, C. (2011). Competition under manufacturer service and retail price. *Economic Modelling, 28*, 1256–1264.

Mukhopadhyay, S. K., Zhu, X., & Yue, X. (2008). Optimal contract design for mixed channels under information asymmetry. *Production and Operations Management, 17*, 641–650.

Pan, K. W., Lai, K. K., Leung, S. C. H., & Xiao, D. (2010). Revenue-sharing versus wholesale price mechanisms under different channel power structures. *European Journal of Operational Research, 203*, 532–538.

Panda, S. (2013). Coordinating two-echelon supply chains under stock and price dependent demand rate. *Asia-Pacific Journal of Operational Research, 30*, 125–151.

Panda, S. (2014). Coordination of a socially responsible supply chain using revenue sharing contract. *Transportation Research Part E: Logistics and Transportation Review, 67*, 92–104.

Paralar, M., & Wang, D. (1993). Diversification under yield randomness in inventory models. *European Journal of Operational Research, 66*, 52–64.

Park, S. Y., & Keh, H. T. (2003). Modeling hybrid distribution channels: A game-theoretic analysis. *Journal of Retailing and Consumer Services, 10*, 155–167.

Ren, L., He, Y., & Song, H. (2014). Price and service competition of dual channel supply chain with consumer returns. *Discrete Dynamics in Nature and Society, 2014*. 10 pp 565603.

Salop, S. C. (1979). Monopolistic competition with outside goods. *The Bell Journal of Economics, 10*, 141–156.

Sarkar, M., & Sarkar, B. (2013). An economic manufacturing quantity model with probabilistic deterioration in a production system. *Economic Modelling, 31*, 245–252.

Sarkar, B. (2015). Supply chain coordination with variable backorder, inspections, and discount policy for fixed lifetime products. *Mathematical Problems in Engineering*.

Shah, N. H., & Shukla, K. T. (2010). Optimal production schedule in declining market for an imperfect production system. *International Journal of Machine Learning and Cybernetics, 1*, 89–99.

Shah, N. H., Gor, A. S., & Jhaveri, C. A. (2012). Optimal pricing and ordering policy for an integrated inventory model with quadratic demand when trade credit linked to order quantity. *Journal of Modelling in Management, 7*, 148–165.

Swaminathan, J. M., & Tayur, S. R. (2003). Models for supply chains in e-business. *Management Science, 49*, 1387–1406.

Tsay, A., & Agrawal, N. (2004). Channel conflict and coordination in the e-commerce age. *Production and Operations Management, 13*, 93–110.

Vinhas, A. S., & Heide, J. B. (2015). Forms of competition and outcomes in dual distribution channels: The distributor's perspective. *Marketing Science, 34*, 160–175.

Wang, X., & Liu, L. (2007). Coordination in a retailer-led supply chain through option contract. *International Journal of Production Economics, 110*, 115–127.

Yao, D. Q., Yue, X. H., Wang, X. Y., & Liu, J. J. (2005). The impact of information sharing on a return policy with the addition of a direct channel. *International Journal of Production Economics, 97*, 196–209.

Zhanga, T., Lianga, L., Yua, Y., & Yu, Y. (2007). An integrated vendormanaged inventory model for a two-echelon system with order cost reduction. *International Journal of Production Economics, 109*, 241–253.

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Please cite this article in press as: Roy, A., et al. Joint decision on EOQ and pricing strategy of a dual channel of mixed retail and e-tail comprising of single manufacturer and retailer under stochastic demand. *Computers & Industrial Engineering* (2016), http://dx.doi.org/10.1016/j.cie.2016.05.002