B.Tech/BT/CE/EE/ME/4th Sem/MATH-2002/2016

2016

Numerical & Statistical Methods
(MATH 2002)

Time Alloted : 3 Hours ***Full Marks : 70***

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : **[10×1=10]**

i) (n+1)th order forward difference of a polynomial of degree n is

(a) n! (b) 0

(c) (n+1)! (d) none of these

ii) If the interval of differencing is unity and $f(x) = ax^2$ (a is constant) which one of the following is wrong?

(a) $\Delta f(x) = a(2x + 1)$ (b) $\Delta^2 f(x) = 2a$

(c) $\Delta^3 f(x) = 2$ (d) $\Delta^4 f(x) = 0$

iii) How many significant figures are there in the decimal number 0.0001234

(a) 7 (b) 4

(c) 8 (d) 6

iv) If A and B are two events with P(A) = 0.4, P(B) = 0.3 and $P(A \cap B) = 0.2$, then $P(A^c \cap B)$ is

(a) 0.1 (b) 0.2

(c) 0.3 (d) 0.4

v) In Simson's $\frac{1}{3}$rd rule, the curve y = f(x) is approximated as

(a) straight line (b) second degree curve

(c) third degree curve (d) fourth degree curve

vi) The variance of first n positive integers is

(a) $\frac{n+1}{2}$ (b) $\frac{n^2+1}{2}$

(c) $\frac{n^2-1}{12}$ (d) 0

vii) A random variable X has the following probability density function :

$$f(x) = \begin{cases} 1 \text{ for } 0 \le x \le 1 \\ 0 \text{ otherwise} \end{cases}$$

The mean of X is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1

viii) If X and Y are independent variates, then correlation coefficient of X and Y is

(a) 1 (b) 0

(c) –1 (d) 2

ix) 50 tickets are serially numbered 1 to 50. One ticket is drawn from these at random. The probability of it being a multiple of 3 or 4 is

(a) $\frac{12}{25}$ (b) $\frac{6}{25}$

(c) $\frac{18}{25}$ (d) 1

x) The mode & median of the observations 4, 6, 8, 6, 7, 8, 8, 4 are

(a) 8 & 7.5 (b) 8 & 8

(c) 7.5 & 7 (d) 8 & 6.5

GROUP - B

2. (a) Find a non zero positive real root of the function sinx + Cosx – 1 = 0 by Regula Falsi method correct to four significant digits.

(b) Solve the following system of linear equations, by LU factorization method.

$2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$

6+6 = 12

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3. (a) Using Newton Raphson method find the value of $\sqrt[4]{12}$, correct to two significant figures.

(b) Solve the following system of linear equations by Gauss Elimination method

$x + 3y + 2z = 5$

$2x - y + z = -1$

$x + 2y + 3z = 2$

6+6 = 12

GROUP - C

4. (a) Find the missing values in the following table :

x	45	50	55	60	65
f(x)	3.0	–	2.0	–	–2.4

(b) Find the solution of the differential equation $\frac{dy}{dx} = x^2 + y$, using modified Euler's method correct up to 5 places of decimal by taking h = 0.01. Given y(0) = 1 for x = 0.02.

5+7 = 12

5. (a) Construct the equivalent polynomial function's expression of the function y = Sinπx taking the points as $x_0 = 0$, $x_1 = \frac{1}{6}$, $x_2 = \frac{1}{2}$

(b) Evaluate $\int_{-1}^{0} xe^x dx$ by using Simson's $\frac{1}{3}$rd rule taking n = 6.

(c) Show that $\Delta^r y_k = \nabla^r y_{k+r}$, where Δ & ∇ are called forward and backward difference operators respectively.

6+4+2 = 12

Group - D

6. (a) An urn contains 10 white 6 black balls. Another urns contains 10 white 8 black balls. Two balls are transferred from the 1st urn to the 2nd urn. Then one ball is taken from the latter. What is the probability that it is a white ball.

(b) Whenever horses a, b and c race together, their respective probabilities of winning are 0.3, 0.5 and 0.2. They race together in three different occasions. Find the probability that the same horse wins all the three races.

(c) X is a random variable. Prove both in the discrete and continuous cases $E(aX + b) = aE(X) + b$

5+4+3 = 12

7. (a) A random variable X has the following p.m.f. :

x	0	1	2	3	4	5	6	7
$P(X=x_1)$	0	k	2k	2k	3k	k^2	$2k^2$	$k+7k^2$

(i) Find the value of K.

(ii) Evaluate $P(X < 6)$, $P(X \geq 6)$ and $P(0 < X < 5)$

(iii) $P(X \leq a) > \frac{1}{2}$ find the minimum value of a.

(b) In an office 40% employees have scooters and 20% have cars. Among those who have scooters 80% do not have cars. What is the probability that an employee has a car given that he does not have a scooter?

(2+3+3)+4 = 12

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GROUP - E

8. (a) Find the probability that none of the three bulbs in a traffic signal will have to be replaced during first 1500 hours of operation if the lifetime X of a bulb is random variable with density $f(x) = \begin{cases} 6(0.25-(x-1.5)^2) & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$

(b) If X is poisson variate such that $P(X = 1) = P(X = 2)$. Find $P(X = 0)$.

(c) A defective die is thrown ten times independently. The probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. What is the probability that odd face appear in each of the ten throws.

6+2+4 = 12

9. (a) For two variables x and y the equations of two regression lines are $x + 4y + 3 = 0$ and $4x + 9y + 5 = 0$. Identify which one is the regression line of y on x. Find the means of x and y. Find the correlation coefficient between x and y. Estimate the values of x when $y = 1.5$.

(b) The length of bolts produced by a machine is normally distributed with mean 4 and standard deviation 0.5. A bolt is defective if its length does not lie in the interval (3.8, 4.3). Find the percentage of defective bolts produced by the machine. Given :

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.6} e^{-\frac{t^2}{2}}dt = 0.7256, \quad \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0.4} e^{-\frac{t^2}{2}}dt = 0.6551$$

(3+2+1+1)+5 = 12