OPERATION RESEARCH AND OPTIMIZATION TECHNIQUES
(MATH 4181)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following Hessian matrices belongs to a concave function?

(a) $\begin{pmatrix} 2 & x \\ 0 & -x^2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & x^2 \end{pmatrix}$

(c) $\begin{pmatrix} -x^2 & x \\ 0 & x \end{pmatrix}$ (d) $\begin{pmatrix} -2 & x \\ -x & -1 \end{pmatrix}$.

(ii) Let $Q(x,y,z)$ be a quadratic form such that $Q(1,1,0) = 2$ and $Q(5,0,0) = -5$, then

(a) $Q(x,y,z)$could be indefinite
(b) $Q(x,y,z)$ could be positive semi definite
(c) $Q(x,y,z)$ could be negative definite
(d) $Q(x,y,z)$could benegative semi definite.

(iii) The point of inflection occurs at $x = x_0$ provided

(a) $f^{(n)}(x_0) = 0$, for n odd (b) $f^{(n)}(x_0) \neq 0$, for n odd
(c) $f^{(n)}(x_0) > 0$, for n even (d) $f^{(n)}(x_0) < 0$, for n even.

(iv) The optimal solution of the following game problem is

		PLAYER B		
		B_1	B_2	B_3
PLAYER A	A_1	3	−2	4
	A_2	−1	4	2
	A_3	2	2	6

(a)(A_3,A_2) (b) (A_3,A_1) (c) (A_2,A_1) (d) (A_1,A_3).

(v) The bordered Hessian matrix of the Lagrange function $L(x,y,\lambda)$is given by

$$H^B\big(L(x,y,\lambda)\big) = \begin{pmatrix} 0 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

then,
(a) the minimum value of the objective function is 0
(b) stationary point is a minimum point
(c) stationary point is a maximum point
(d) stationary point is a saddle point.

(vi) The range of values of p and q for which 2 is the value of the following game:

Player B

Player A

0	2	3
8	5	q
2	p	4

(a) $p \geq$ & $q \geq 5$ (b) $p \geq 5$ & $q \leq 5$
(c) for any value of p & q (d) $p \leq -5$ & $q \geq 5$.

(vii) The basic feasible solutions of the system

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 + x_2 + 4x_3 = 4$$

are
(a) $\left(0, \frac{1}{5}, \frac{2}{5}\right)$, $(-6, 0, 4)$ and $(\frac{2}{3}, \frac{8}{3}, 0)$
(b) $\left(0, -\frac{1}{5}, \frac{2}{5}\right)$, $(6, 0, 4)$ and $(\frac{2}{3}, \frac{8}{3}, 0)$
(c) $(6, 0, 4)$ and $(\frac{2}{3}, \frac{8}{3}, 0)$
(d) $\left(0, \frac{1}{5}, \frac{2}{5}\right)$ and $\left(\frac{2}{3}, \frac{8}{3}, 0\right)$.

(viii) A function is unimodal in $a \leq x \leq b$ if
(a) it contains a unique optimal solution
(b) it contains two optimal solution
(c) it is a constant function
(d) it is not a monotonic function.

(ix) If the basis contains one or more artificial variables at positive level, the original problem will have
(a) degenerate solution (b) infinitely many solution
(c) unbounded solution (d) no feasible solution.

(x) Golden section search method is used for solving
(a) Linear programming problem
(b) Non-linear programming problem
(c) Travelling salesman problem
(d) Assignment problem.

Group – B

2. (a) Egg contains 6 unit of vitamin A per gram and 7 unit of vitamin B per gram and costs 12 paisa per gram. Milk contains 8 unit of vitamin A per gram and 12 unit of vitamin B per gram and costs 20 paisa per gram. The daily minimum requirements for vitamin A and vitamin B are 100 unit and 120 unit. Find the minimum amount of egg and milk. Formulate the L.P.P. and find the optimal solution by graphical method.

(b) Solve the following L.P.P. using Big-M method:
Maximize $z = 3x_1 - x_2$
subject to the constraints

$$2x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_2 \leq 4$$
$$x_1,\ x_2 \geq 0.$$

5 + 7 = 12

3. (a) Solve the following L.P.P. by Simplex method:
Maximize $z = 5x_1 + 3x_2$
subject to the constraints

$$x_1 + x_2 \leq 2$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12$$
$$x_1,\ x_2 \geq 0.$$

(b) Find the dual of the following L.P.P.:
Maximize $z = x_1 + 4x_2 + 3x_3$
subject to
$2x_1 + 3x_2 - 5x_3 \leq 2$
$3x_1 - x_2 + 6x_3 \geq 1$
$x_1 + x_2 + x_3 = 4$
$x_1,\ x_2 \geq 0$; x_3 is unrestricted in sign.

7 + 5 = 12

Group – C

4. (a) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	D_4	Supply
O_1	2	2	2	1	3
O_2	10	8	5	4	7
O_3	7	6	6	8	5
Demand	4	3	4	4	

(b) Find the optimal assignment from the following profit matrix:

	1	2	3	4
A	40	35	43	45
B	33	39	48	33
C	40	37	33	32
D	35	41	39	37

7 + 5 = 12

5. (a) Use dominance to reduce the following pay-off matrix to a 2 × 2 game and hence find the optimal strategies and the value of the game:

PLAYER B

PLAYER A

3	5	4	2
5	6	2	4
2	1	4	0
3	3	5	2

(b) Use graphical method in solving the following game and find the value of the game.

PLAYER B

PLAYER A

0	−2
7	−1
−1	4
−2	6
5	−3

6 + 6 = 12

Group – D

6. (a) Use the method of Lagrangian multipliers to solve the following non-linear programming problem. Does the solution maximize or minimize the objective function?

Optimize $Z = 2x_1^2 + x_2^2 + 3x_3^2 + 10x_1 + 8x_2 + 6x_3 - 100$

Subject to the constraint

$$x_1 + x_2 + x_3 = 20$$
$$x_1, x_2, x_3 \geq 0.$$

(b) Verify whether the following function is convex or concave and find the maximum or minimum solution point:
$$f(x_1, x_2, x_3) = 4x_1^2 + 3x_2^2 + x_3^2 - 6x_1x_2 + x_1x_3 - \frac{x_1}{2} - 2x_2 + 15.$$

8 + 4 = 12

7. Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Maximize $z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$
subject to the constraints
$$x_1 + x_2 \leq 1$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1, x_2 \geq 0.$$

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Group – E

8. Minimize the function $f(x) = 2 - 4x + e^x$ in the interval $[0.5, 2.5]$ with an accuracy of $\epsilon = 0.002$ using Dichotomous Search algorithm. Take the tolerance value to be less than 0.3.

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9. Find the minimum of the function $f(x) = x^5 - 5x^3 - 20x + 5$ by interval halving method in the interval $[0, 5]$. Perform 4 iterations.

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Department & Section	Submission Link
CSE	https://classroom.google.com/c/MTQxNDg1MTgyNDE2/a/MjcxNjI0MzcwNDg2/details
ECE	https://classroom.google.com/c/MTUxNTY5Mzc0NTQy/a/Mjc0MDM0MTQxNTY0/details
IT	https://classroom.google.com/c/MTE5MDg5MDA2MjY1/a/MjY0NzYwMDU2MzY0/details

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