NUMERICAL METHODS OF ANALYSIS
(CHEN 3104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The number 88.538 is chopped to the 2nd decimal place The percent true error due to chopping is
(a) 0.008 (b) 0.000096
(c) 0.0096 (d) 0.008.

(ii) An algorithm is said to be stable if
(a) Results change slightly with perturbations in the independent variable
(b) Results change grossly with perturbations in the dependent variable
(c) Results do not change with any perturbation
(d) Results decay over time in time-dependent problem.

(iii) A condition for Gauss-Siedel method to converge iteratively is
(a) Diagonal terms should be nonzero
(b) Diagonal terms greater than the maximum of the off-diagonal terms
(c) Diagonal terms greater than the sum of the off-diagonal terms
(d) No off-diagonal terms to be present.

(iv) The following algorithms are direct
(a) Gauss Siedel (b) Gauss Jordon
(c) Gauss Elimination (d) b and c.

(v) The zeroth order approximation for the expansion of e^x for x = 1 is
(a) 3 (b) 2
(c) 1 (d) 0.

(vi) Heun's method is a special form of
(a) Euler's method (b) Jacobi method
(c) 2nd order RK method (d) None of above.

(vii) Two point boundary value problems usually contain
(a) First derivative in the unknown variable
(b) Second derivative in the unknown variable
(c) One boundary conditions
(d) One boundary and one initial condition.

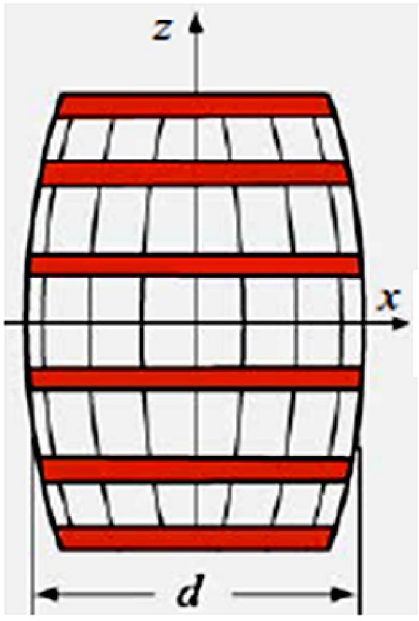
(viii) The Euclidean norm of a vector $\underline{x}$ with n components is
(a) Dependent on the magnitude of each component
(b) Is dependent on the direction of $\underline{x}$
(c) Dependent on the square of each component
(d) None of above.

(ix) Parabolic PDEs are obtained by mathematical modelling of
(a) Heat conduction along a thin long rod
(b) Heat convection along two dimensions
(c) Unsteady heat conduction in a rectangular slab
(d) Steady heat conduction in a rectangular slab.

(x) The eigen values of $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ are
(a) Imaginary
(b) Real and equal
(c) Real and unequal
(d) zero.

Group – B

2. (a) [GS Pr 9.2]
To estimate the surface area of a drum, diameter of the drum is measured at various points on the drum.



z (inch)	-12	-6	0	6	12
d (inch)	0	3.2	4.4	5.6	6

Find the radius of the drum at z = -8.
What kind of interpolation did you use and why?

(b) State two different types of interpolation possible. Explain the pros and cons of both.

6 + 6 = 12

3. (a) The following experimental data is provided:

x	1	2	3	5	8
y^2	0.64	3.61	4.84	9	12.25

Determine the lagrange polynomial relating y^2 with x. Determine y(2.5).

(b) State two different types of numerical integration used. Which is better and why?

6 + 6 = 12

Group – C

4. (a) Solve the following linear system using Gauss – Siedel iterative method.

$$4x_1 - 2x_2 - 3x_3 + 6x_4 = 12$$
$$-6x_1 + 7x_2 + 6.5x_3 - 6x_4 = -6.5$$
$$x_1 + 7.5x_2 + 6..25x_3 + 5.5x_4 = 16$$
$$-12x_1 + 22x_2 + 15.5x_3 - x_4 = 17$$

(b) What is the difference between Gauss-Siedel method and Gauss Elimination?

9 + 3 = 12

5. (a) Determine the root of $f(x) = x - 2e^x$. Using the bisection method. Start with a= 0 and b= 1, and carry out the first three iterations.

(b) State four problems where Newton-raphson method of root finding might fail.

8 + 4 = 12

Group – D

6. (a) Consider the following first order ODE: $\frac{dy}{dx} = 1000 * (\frac{x^2}{y})$ from with y(0) = 2.

Using Euler's method, evaluate y(0.2) with at step size of 0.1.

(b) Integrate the system analytically and evaluate the true error expressed as percentage.

8 + 4 = 12

7. (a) Consider the following ordinary differential equation:

$$\frac{dy}{dt} = y + t^3$$

Integrate the system from t = 0 to t = 0.5 with initial condition y(0) = 1. Use Heun's method and a step size of 0.5.

(b) Why would you use a high order Runge Kutta method for solving ODE when Euler method is easier to solve?

8 + 4 = 12

Group – E

8. (a) Gas species A is diffusing though a plug flow reactor from x = 0 to x = 10 cm. The concentration of A can be modelled using the following PDE. $\frac{\partial C_A}{\partial t} = 0.5\frac{\partial^2 C_A}{\partial x^2}$

Subject to the boundary conditions

$C_A(0,t) = 1$
$C_A(10,t) = 0$
$C_A(x,0) = 0 \text{ for all } 0 < x < 10$

Choose two intermediate grid points along the x>0 direction. Setup the numerical form of the differential equation in form $\underline{\underline{A}}\underline{y} = \underline{B}$ using an implicit scheme. Clearly write out $\underline{\underline{A}}$ and $\underline{B}$.

(b) What is a Robin boundary condition?

10 + 2 = 12

9. (a) The following differential equation is to be solved over a domain, 0<x<1 and 0<y<1 subject to boundary conditions given below.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 5$$
$$x = 0 \text{ for all y}, u = 1$$
$$x = 0.5 \text{ for all y}, \frac{du}{dx} = 10$$
$$y = 0 \text{ for all x}, u = 0$$
$$y = 0.8 \text{ for all x}, u = 0$$

What kind of partial differential equation is represented by the above equation? What are the dependent and independent variable? For each boundary condition, indicate the type of boundary condition used.

Divide the domain with two interior grid points in the x and y direction What is the dimension of the grid size in the x and y direction?

(b) Write out the numerical form of the difference equation.

5 + 7 = 12

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CHE	https://classroom.google.com/c/MTIyMDYwODU0NTU1/a/MjgyNTM0MTI0OTky/details