OPTIMIZATION TECHNIQUES
(MATH 6121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If in a system $AX = b$, one of the basic variables become zero, the solution is said to be
(a) a non-degenerate basic solution (b) a degenerate basic solution
(c) a feasible solution (d) a non-feasible solution.

(ii) The LPP Min $z = x_1 + x_2$
Subject to
$x_1 + 2x_2 \leq 10$
$2x_1 + 2x_2 \geq 3$
$x_1, x_2 \geq 0$
has
(a) a unique solution (b) no solution
(c) infinitely many solutions (d) unbounded solution.

(iii) North-west corner method is used to find
(a) an optimal solution of a transportation problem
(b) an IBFS of an LPP
(c) a solution of an assignment problem
(d) the IBFS of a transportation problem.

(iv) If the i^{th} constraint of the primal problem is an equality, then
(a) i^{th} variable of the dual is non negative in sign
(b) i^{th} variable of the dual is negative in sign
(c) i^{th} variable of the dual is unrestricted in sign.
(d) i^{th} constraint of the dual is an equality.

(v) The price of an artificial variable in an objective function is
(a) a very large negative number (b) a very large positive number
(c) 0 (d) -1.

(vi) Given the optimization problem
Optimize $f(x, y, z, w)$ subject to $g_i(x, y, z, w) = b_i, i = 1,2$; then the order of the bordered Hessian matrix of the Lagrangian function is
(a) 4×4 (b) 4×6 (c) 6×4 (d) 6×6.

(vii) The range of p for which the following payoff matrix is strictly determinable is

PLAYER B

PLAYER A

p	6	2
-1	p	-7
-2	4	p

(a) $p \geq -1$ (b) $p \leq 2$
(c) $-1 \leq p \leq 2$ (d) for any value of p.

(viii) The Hessian matrix of function f(x, y) is given by
$\text{Hf}(x, y) = \begin{pmatrix} -2 & x \\ -x & -1 \end{pmatrix}$
If $(-1, 1)$ is a stationary point, then this point would be
(a) a local minimum but not global minimum point
(b) a global maximum point
(c) a saddle point
(d) a local maximum but not global maximum point.

(ix) Let $Q(x, y, z)$ be a quadratic form such that $Q(x, y, z) > 0$ for $(x, y, z) \neq 0$ and $Q(0, 0, 0) = 0$, then
(a) $Q(x, y, z)$ could be indefinite
(b) $Q(x, y, z)$ could be positive definite
(c) $Q(x, y, z)$ could be negative definite
(d) $Q(x, y, z)$ could be positive semi definite.

(x) Which of the following Hessian matrices belongs to a concave function?
(a) $\begin{pmatrix} -2 & x \\ 0 & -x^2 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & x^2 \end{pmatrix}$
(c) $\begin{pmatrix} -x^2 & x \\ 0 & -x \end{pmatrix}$ (d) $\begin{pmatrix} -2 & x \\ x & -1 \end{pmatrix}$.

Group – B

2. (a) Food X contains 6 units of Vitamin A per gram and 7 units of Vitamin B per gram and costs 12 paise per gram. Food Y contains 8 units of Vitamin A per gram and 12 units of Vitamin B and costs 20 paise per gram. The daily minimum requirements of Vitamins A and B are 100 units and 120 units respectively. Formulate the given problem as an LPP and solve using graphical method to find the minimum cost of the product units. [(CO1, CO2)(Create/HOCQ)]

(b) Use Simplex method to solve the following linear programming problem:
Maximize $z = 3x_1 + 2x_2$
Subject to

$x_1 - x_2 \leq 5$
$2x_1 - 3x_2 \leq 30$ [(CO1, CO2)(Apply/IOCQ)]
$x_1, x_2 \geq 0$

6 + 6 = 12

3. (a) Solve the following linear programming problem using 'Big-M' method:
Maximize $z = 3x_1 + 2x_2$
Subject to
$2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12$ [(CO1, CO2) (Apply/IOCQ)]
$x_1, x_2, x_3 \geq 0$

(b) Obtain the dual of the following LPP:
Maximize $z = 2x_1 + 3x_2 + x_3$
Subject to
$4x_1 + 3x_2 + x_3 = 6$
$x_1 + 2x_2 + 5x_3 = 4$ [(CO1, CO2) (Understand/LOCQ)]
$x_1, x_{2,} x_3 \geq 0$

7 + 5 = 12

Group – C

4. (a) Solve the transportation problem by Vogel's approximation method and checking it's optimality, find the optimal solution:

	D_1	D_2	D_3	D_4	Supply
O_1	15	10	17	18	2
O_2	16	13	12	13	6
O_3	12	17	20	11	7
Demand	3	3	4	5	

[(CO1, CO2, CO3)(Evaluate/HOCQ)]

(b) Determine the IBFS of the following transportation problem by Matrix-Minima method.

	D_1	D_2	D_3	D_4	Supply
O_1	19	20	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	

[(CO1, CO2, CO3) (Evaluate/HOCQ)]

8 + 4 = 12

5. (a) Write the mathematical formulation of an assignment problem.
[(CO1, CO2, CO3) (Understand/LOCQ)]

(b) A departmental head has three tasks 1, 2 and 3 to be performed by his four subordinates A, B, C and D. The subordinates differ in efficiency. The estimates of the time, each subordinate would take to perform, is given in the following matrix. How should he allocate tasks to each man so as to minimize the total man hours?

	1	2	3
A	9	26	15
B	13	27	6
C	35	20	15
D	18	30	20

[(CO1, CO2, CO3)(Evaluate/HOCQ)]

6 + 6 = 12

Group – D

6. (a) Use graphical method in solving the following game and find the value of the game.

	PLAYER B	
PLAYER A	2	4
	2	3
	3	2
	−2	6

[(CO1, CO4) (Understand/LOCQ)]

(b) Use algebraic method to solve the following game:

	PLAYER B		
PLAYER A	1	−1	0
	−1	1	1
	2	−1	−2

[(CO1, CO4) (Apply/IOCQ)]

6 + 6 = 12

7. (a) Find the solution of the game whose payoff matrix is given below:

	PLAYER B				
PLAYER A	−4	−2	−2	3	1
	1	0	−1	0	0
	−6	−5	−2	−4	4
	3	1	−6	0	−8

[(CO1, CO4) (Understand/LOCQ)]

(b) Use dominance to reduce the following pay-off matrix to a 2×2 game and hence find the optimal strategies and the value of the game:

	PLAYER B			
PLAYER A	3	2	4	0
	3	4	2	4
	4	2	4	0
	0	4	0	8

[(CO1, CO4) (Apply/IOCQ)]

4 + 8 = 12

Group – E

8. (a) Use Kuhn-Tucker conditions to solve the following non-linear programming problem:

Minimize $z = (x_1 - 2)^2 + (x_2 - 1)^2$

Subject to the constraints

$x_1^2 - x_2 \leq 0$

$x_1 + x_2 - 2 \leq 0$

$x_1, x_2 \geq 0$ [(CO5, CO6) (Evaluate/HOCQ)]

(b) Determine the relative maximum and minimum (if any) of the following function:

$f(x_1, x_2, x_3) = x_1 + x_1 x_2 + 2x_2 + 3x_3 - x_1^2 - 2x_2^2 - x_3^2$

[(CO5, CO6) (Understand/LOCQ)]

8 + 4 = 12

9. Solve the following non-linear programming problem, using the Lagrangian multipliers method:

Optimize $Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1 x_2$

Subject to the constraints

$x_1 + x_2 + x_3 = 15$

$2x_1 - x_2 + 2x_3 = 20$

$x_1, x_2, x_3 \geq 0.$ [(CO5, CO6) (Evaluate/HOCQ)]

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Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	10.41%	43.75%	45.83%

Course Outcome (CO):

After the completion of the course students will be able to:

MATH6121.1 Describe the way of writing mathematical model for real-world optimization problems.

MATH6121.2 Identify Linear Programming Problems and their solution techniques.

MATH6121.3 Categorize Transportation and Assignment problems.

MATH6121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.

MATH6121.5 Convert practical situations into non-linear programming problems.

MATH6121.6 Solve unconstrained and constrained programming problems using analytical techniques.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	**Submission link:**
CSE	https://classroom.google.com/c/NDA0Nzc5NDUwNTAz/a/NDYzOTgwNTgzNzYx/details