ADVANCED DISCRETE MATHEMATICS AND STATISTICAL METHODS
(MATH 5101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The probability of drawing a king, or a diamond, or both from a well shuffled pack of 52 cards is
(a) $\frac{1}{13}$ (b) $\frac{4}{13}$ (c) $\frac{3}{13}$ (d) $\frac{3}{52}$.

(ii) The total number of ways such that 6 boys and 4 girls sit in a row is
(a) 10! (b) 6! (c) 4! (d) $\frac{6!}{4!}$.

(iii) If X is normally distributed with mean zero and unit variance, then the expectation of X^2 is
(a) 1 (b) 2 (c) 8 (d) 0.

(iv) $x^4 - 3x^3 + 2x^2 + x - 1$ is
(a) the chromatic polynomial of a complete graph
(b) the chromatic polynomial of a tree
(c) the chromatic polynomial of a bipartite graph
(d) not the chromatic polynomial of any graph.

(v) The harmonic mean (H.M.) of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ is
(a) $\frac{25}{12}$ (b) $\frac{25}{48}$ (c) $\frac{48}{25}$ (d) $\frac{2}{5}$.

(vi) The total number of ways such that the integers 1 through 9 be permuted such that no odd integer will be in its natural position is
(a) 205045 (b) 205050 (c) 205056 (d) 205060.

(vii) If $F(x)$ is the distribution function of a discrete random variable, then
(a) $F(x)$ is continuous everywhere
(b) $F(-\infty) = -1$
(c) $F(\infty) = 1$
(d) $F(x)$ is monotonically decreasing function.

(viii) A disconnected planar graph has 6 edges, 10 vertices and 3 faces. The number of components in the graph is
(a) 5 (b) 6 (c) 7 (d) 8.

(ix) The generating function for the sequence 1, 2, 3, 4, … is given by
(a) $\frac{1}{(1-x)^3}$ (b) $\frac{1}{(1-x)^2}$ (c) $\frac{1}{1-x}$ (d) $\frac{x}{1-x}$.

(x) When two variables x, y are uncorrelated, then the correlation coefficient between them is
(a) 0 (b) ± 1 (c) 1 (d) -1.

Group- B

2. (a) If A and B are two independent events, then show that
(i) $\bar{A}$ and $\bar{B}$ are independent,
(ii) $\bar{A}$ and B are independent. [(CO1, CO2)(Remember/LOCQ)]

(b) The probability density function of a random variable X is
$f(x) = \begin{cases} k(x-1)(2-x), \text{ for } 1 \le x \le 2 \\ 0, \quad \text{elsewhere.} \end{cases}$
Determine
(i) the value of k
(ii) the distribution function $F(x)$
(iii) $P\left(\frac{5}{4} \le X \le \frac{3}{2}\right)$. [(CO1, CO2) (Apply/IOCQ)]

6 + 6 = 12

3. (a) Two players A and B alternatively throw a pair of dice. A wins if A throws 6 before B throws 7, and B wins if B throws 7 before A throws 6. If A begins, then find the probability that A wins. [(CO1, CO2) (Understand/LOCQ)]

(b) The demand for a new product of a company is assumed to be a continuous random variable with the following probability density function $f(x) = \begin{cases} \frac{x}{a^2} e^{-\frac{x^2}{2a^2}}, & x \ge 0 \\ 0, & x < 0 \end{cases}$. Find the mean and variance of this random variable, and also find the probability that the demand will exceed a (> 0).
[(CO1, CO2) (Apply/IOCQ)]

6 + 6 = 12

Group – C

4. (a) The marks obtained by 1000 students in a final examination are found to be approximately normally distributed with mean 70 and standard deviation 5. Estimate the number of students whose marks will be between 60 and 75, both inclusive. Given that area under the normal curve $\Phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$ between $z = 0$ and $z = 2$ is 0.4772 and between $z = 0$ and $z = 1$ is 0.3413.
[(CO1, CO2) (Apply/IOCQ)]

(b) A random variable follows binomial distribution with mean 4 and standard deviation $\sqrt{2}$. Find the probability of assuming non-zero value of the variable.

[(CO1, CO2) (Apply/IOCQ)]

6 + 6 = 12

5. (a) Calculate the median and mode of the following distribution:

Heights (inches)	$60-64$	$65-69$	$70-74$	$75-79$	$80-84$	$85-89$
Frequency	8	28	118	66	16	8

[(CO1, CO2) (Evaluate/HOCQ)]

(b) From the following data, find the coefficient of linear correlation between X and Y. Determine also the regression line of Y on X.

X	1	3	4	6	8	9	11	14
Y	1	2	4	4	5	7	8	9

[(CO1, CO2) (Evaluate/HOCQ)]

6 + 6 = 12

Group - D

6. (a)
(i) How many 5-card hands consist only of hearts?
(ii) How many 5-card hands consist of cards from a single suit?
(iii) How many 5-card hands have 2 clubs and 3 hearts?
(iv) How many 5-card hands have 2 cards of one suit and 3 cards of a different suit?
(v) How many 5-card hands contain 2 aces and 3 kings?
(vi) How many 5-card hands contain exactly 2 of one kind and 3 of another kind? [(CO1, CO2, CO3) (Evaluate/HOCQ)]

(b) In a survey of students at Florida State University the following information was obtained. 260 were taking a statistics course, 208 were taking a mathematics course, 160 were taking a computer programming course, 76 were taking statistics and mathematics, 48 were taking statistics and computer programming, 62 were taking mathematics and computer programming, 30 were taking all 3 kinds of courses and 150 were taking none of the 3 courses.
(i) How many students were surveyed?
(ii) How many students were taking a statistics and a mathematics course but not a computer programming course?
(iii) How many students were taking a statistics and a computer programming course but not a mathematics course?
(iv) How many were taking a computer programming and a mathematics course but not a statistics course?
(v) How many were taking a statistics course but not a course in mathematics or computer programming?
(vi) How many were taking a mathematics course but not a course in statistics or computer programming? [(CO1, CO2, CO3)(Analyze/IOCQ)]

6 + 6 = 12

7. (a) If m is an odd positive integer, prove that there exists a positive integer n such that m divides $(2^n - 1)$. [(CO1, CO2, CO3) (Create/HOCQ)]

(b) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} + 3n.2^n,\ n \geq 1$, given that $a_0 = 4$. [(CO1, CO2, CO3) (Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) Let G be a connected planar 4-regular graph with 10 faces. Determine how many vertices does G have. [(CO1, CO2, CO4) (Analyse/IOCQ)]

(b) Find the vertex connectivity and a cut set of the graph given in Figure 1.

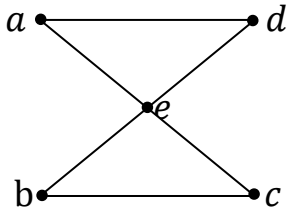


Figure 1

[(CO1, CO2, CO4)(Understand/LOCQ)]

(c) Suppose that three applicants A, B and C apply for five vacant positions P, Q, R, S and T in a company. Each person is qualified for one or more jobs. The following information is available from the applications:

Applicants	Positions for their qualification
A	Q, S, T
B	P, Q, R
C	P, Q, R, S

(i) Draw the bipartite graph for the relationship.

(ii) Using Hall's condition, check whether each applicant can get a suitable vacant position.

(iii) Give two solutions of this assignment problem.

[(CO1,CO2,CO4)(Evaluate/HOCQ)]

4 + 2 + 6 = 12

9. (a) Using Decomposition Theorem find the chromatic polynomial of the graph G given in Figure 2. Also, find the chromatic number of G using its chromatic polynomial.

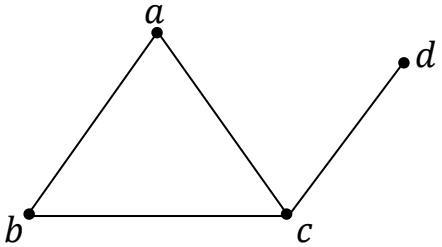


Figure 2

[(CO1, CO2, CO4) (Apply/IOCQ)]

(b) Take W_4 (wheel graph with 4 vertices) and K_3 (complete graph with 3 vertices) and join any one vertex of K_3 with each vertex of W_4.

(i) Sketch the resulting graph.

(ii) Find a maximum matching in the resulting graph. Is this maximum matching, a perfect matching as well?

(iii) Find the clique number and the chromatic number of the resulting graph, with justification. [(CO1, CO2, CO4) (Create/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	17.6%	47.1%	35.3%

Course Outcome (CO):

After the completion of the course students will be able to:

MATH 5101.1. To understand the mathematical fundamentals that is prerequisites for a variety of courses like Data mining, Network protocols, analysis of Web traffic, Computer security, Software engineering, Computer architecture, operating systems, distributed systems, Bioinformatics, Machine learning.

MATH 5101.2. To develop the understanding of the mathematical and logical basis to many modern techniques in information technology like machine learning, programming language design, and concurrency.

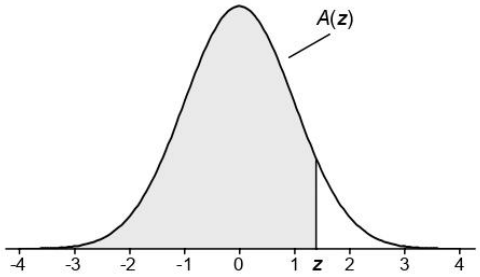
MATH 5101.3. To study the principles of enumeration.

MATH 5101.4. To equip oneself with the techniques used in graph theory.

Department & Section	Submission Link
CSE	https://classroom.google.com/c/NDAxNjQ3NTAwMDEw/a/NDY0MjQ5NTkzMjg3/details

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							