B.TECH/ME/7TH SEM/MECH 4128/2021

COMPUTATIONAL METHODS IN ENGINEERING
(MECH 4128)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) During computation, with increase of step size [CO1 (Remember/LOCQ)]
(a) Round off error increases (b) Truncation error decreases
(c) Truncation error increases (d) both the error decreases.

(ii) The number of significant digits in 0.0099 is [CO1 (Recognize/LOCQ)]
(a) 4 (b) 5 (c) 2 (d) 3.

(iii) In which method, the constant coefficient matrix will be unaltered during row operation [CO1 (Classify/LOCQ)]
(a) Gauss Elimination (b) LU decomposition
(c) Both (a) and (b) (d) Matrix inversion.

(iv) In LU decomposition method [CO2 (Remember /LOCQ)]
(a) L and U matrices can be found by two different calculation
(b) L can be automatically found if U is calculated
(c) U can be automatically found if L is calculated
(d) L and U need not to be found under any circumstances.

(v) A non-polynomial equation can be estimated by [CO3 (Understand/LOCQ)]
(a) Linear regression (b) nonlinear regression
(c) polynomial regression (d) multiple linear regression.

(vi) In polynomial regression of order n, number of regression equations are [CO2 (Classify/LOCQ)]
(a) n+1 (b) n (c) n-1 (d) n+2.

(vii) In Simpson's 1/3rd rule, the result of integration is equal to sum of the functional values multiplied with [CO4 (Remember/LOCQ)]
(a) h/2 (b) h/3 (c) 3h/8 (d) h/4.

(viii) Which of the following statement is incorrect about numerical methods? [CO5 (Describe/LOCQ)]

(a) Numerical methods can be used to solve any differential equation
(b) Analytical solutions of differential equations are generally very difficult to obtain and hence numerical methods are used.
(c) Solutions obtained by numerical methods are always exact solutions.
(d) The domain of problem is discretized in finite number of grids and the solution is obtained at the nodal points.

(ix) The order and degree of the ordinary differential equation $\frac{d^4y}{dx^4} + 3\left(\frac{dy}{dx}\right)^5 + 5y = 0$ are respectively [CO6 (Classify/LOCQ)]
(a) 1; 5 (b) 4; 5 (c) 4; 1 (d) 5; 1.

(x) Which of the following statement is true about PDE $\frac{\partial^2 U}{\partial x^2} - \frac{\partial^2 U}{\partial y^2} - \frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = 0$: [CO5 (Identify/LOCQ)]
(a) It is parabolic in nature (b) It is hyperbolic in nature
(c) It is an instance of Laplace equation (d) It is an instance of Poisson equation.

Group – B

2. (a) Newton's law of cooling says that the temperature of a body changes at a rate proportional to the difference between its temperature and that of the surrounding medium.

$$\frac{dT}{dt} = k(T - T_a)$$

Where T = the temperature of the body (^{0}C), t = time (min), k is proportionality constant (per minute) and T_a = the ambient temperature (^{0}C). Suppose that a cup of coffee originally has a temperature of 85^{0}C. Use Euler's numerical method to compute the temperature from t =0 to 10 min, using a step size of 2 min. Take T_a = 25^{0}C and k = 0.015 /min. [(CO1) (Apply/IOCQ)]

(b) Write the function of significant digits in numerical analysis. [(CO1) (Classify/LOCQ)]

8 + 4 = 12

3. (a) Use Gauss Elimination process to solve the following set of linear equations. Find out the digits up to 2 decimal places accurately. [(CO2) (Solve/IOCQ)]

$$10x + 2y - z = 27$$
$$3x + 5y - 2z = 61.5$$
$$x + y + 6z = -21.5$$

(b) Use LU Decomposition process to solve the following set of linear equations. [(CO2) (Solve/IOCQ)]

$$8x + 2y - 2z = -2$$
$$10x + 2y + 4z = 4$$
$$12x + 2y + 2z = 6$$

6 + 6 = 12

Group – C

4. (a) Dynamic viscosity of water is related to temperature in the following manner.

Temperature (^{0}C)	0	5	10	20	30	40
Dynamic Viscosity (Pa-S)	1.79	1.52	1.31	1.02	0.79	0.65

[(CO3) (Implement/IOCQ)]

(b) Using linear regression find out the equation of a straight line which is best fit to the following data points. Here $q = f(p)$ [(CO3) (Apply/IOCQ)]

p	0	2	3	4	5	6
q	1	6	5	8	12	11

7 + 5 = 12

5. (a) Six different values of stresses for fracture times are plotted during an experiment. Predict the value of fracture time for an applied stress of 17 kg/mm^2 using Lagrange interpolating polynomial. [(CO3) (Solve/IOCQ)]

Applied Stress (kg/mm^2)	5	10	15	20	25	30
Fracture time (hr.)	40	30	25	40	18	20

(b) Using linear regression find out the equation of a straight line which is best fit to the following data points. [(CO3) (Solve/IOCQ)]

x	5	7	8	11	15	20
y	20	31	36	45	66	87

7 + 5 = 12

Group – D

6. (a) The explicit form of error function is written below.

$$\boldsymbol{erf\ (a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-x^2} dx}$$

Use two point Gauss quadrature to estimate erf (1.5). Find the relative percentage error if the exact value of erf (1.5) = 0.966105.
[(CO4) (Implement/IOCQ)]

(b) Using Trapezoidal rule, evaluate the following integral with 4 intervals.

$$\int_0^3 (5 + 3cos\ x)\ dx$$

[(CO4) (Implement/IOCQ)]

6 + 6 = 12

7. (a) Use the Taylor series expansion to evaluate the integral of $y' - 2y = 3e^x$; $y(0) = 0$ at (i) x = 0.1 (0.1) 0.3 (ii) x = 1.0; 1.1 [Retain terms up to x^5]. [(CO5) (Implement/IOCQ)]

(b) Evaluate the initial value problem $y' - y^2 = 0$; $y(0) = 1$ by the Euler's method at x = 0.1 and x = 0.2. [(CO5) (Formulate/HOCQ)]

(5 + 2) + 5 = 12

Group – E

8. (a) Match the following:

(I) 2 − D Laplace equation: $u_{xx} + u_{yy} = 0$	**(A)** Parabolic
(II) 1 − D wave equation: $u_{tt} = c^2 u_{xx}$	**(B)** Hyperbolic

(III) $1-$ D transient heat conduction equation $u_t = c^2 u_{xx}$	**(C)** Elliptic

[(CO6) (Classify/LOCQ)]

(b) Use the classical 4th order Runge-Kutta method to find the solution of the IVP $\boldsymbol{y' - y^2 = 0;\ y(0) = 1}$ on [0,1].
Show the calculations clearly in tabular form till the 5th iteration step. Take $h = 0.2$. [(CO5) (Implement/IOCQ)]

3 + 9 = 12

9. (a) Solve the initial boundary-value problem $\boldsymbol{\frac{\partial f}{\partial t} = 2\frac{\partial^2 f}{\partial x^2};\ 0 \le x \le 6, t > 0}$ by the Schmidt explicit method.
The two boundary conditions are $f(x = 0, t) = 10$ and $f(x = 6, t) = 18$. The initial condition is $f(x, t = 0) = x^2/2$. Consider the spatial increment $h = 1$ and the time increment $k = 1/8$.
Compute the field values for $0 \le t \le 1$. Estimate the field values as $t \to \infty$. [(CO6) (Formulate/HOCQ)]

(b) Find the condition for the second order PDE $\boldsymbol{u_{xx} + 2xu_{xy} + (1 - y^2)u_{yy} = 0}$ to be hyperbolic. [(CO6) (Apply/IOCQ)]

9 + 3 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	16.9%	69.8%	13.2%

Course Outcome (CO):
After the completion of the course students will be able to
CO 1: Apply different mathematical models to obtain numerical solutions and classify different types of error.
CO 2: Analyze and solve a system of linear algebraic equations by different methods and find out the roots.
CO 3: Implement the regression and interpolation methods for curve fitting and apply different types of optimization techniques to solution of problems.
CO 4: Use different numerical integration methods for practical problems.
CO 5: Classify Initial-value and Boundary-value problems in order to formulate their solutions, implement different methods for their solutions, and solve Eigen value problems applied to physical systems.
CO 6: Classify linear, second-order partial differential equations (PDEs) as elliptic, parabolic, or hyperbolic, and apply the Finite Difference Method to formulate the solutions of different classes of PDEs.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission link:
ME	https://classroom.google.com/c/NDA1NjAxNjQ0ODc3/a/NDYzODI2NTM0MjE5/details