PHYSICS - II
(PHYS 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The number of degrees of freedom of a rigid body which has one point fixed but can move in space about this point is
(a) 6 (b) 3 (c) 1 (d) 0.

(ii) Three point masses $m, 2m$ and m are located at $(0, a, a), (a, a, a)$ and $(3a, a, 0)$ respectively. The value of I_{xy} is
(a) $5ma^2$ (b) $-10ma^2$ (c) $-5ma^2$ (d) $-7ma^2$

(iii) The cyclic coordinate in the Lagrangian $L = \frac{1}{2}m\dot{q}_1{}^2 + \frac{1}{2}Mq_1{}^2\dot{q}_2{}^2 + m\dot{q}_3{}^2 - V(q_2, q_3)$ is
(a) q_2 (b) q_3 (c) q_1 (d) none

(iv) A coupled two-particle-system has normal mode frequencies ω_1 and ω_2. The ratio between normal frequencies for which the system is not periodic is
(a) $1:2$ (b) $2:1$ (c) $1:\sqrt{2}$ (d) $1:10$.

(v) A small oscillating system has two normal coordinates given by $Q_1 = \frac{1}{\sqrt{2}}(q_1 + q_2)$ and $Q_2 = \frac{1}{\sqrt{2}}(q_1 - q_2)$ corresponding to the normal mode frequencies ω_1 and ω_2 respectively. The system oscillates simple harmonically with frequency ω_1 when
$(a) Q_1 = 0$ (b) $Q_2 = 0$ (c) $Q_1 = 2Q_2$ (d) $Q_2 = 2Q_1$

(vi) The kinetic energy matrix of a coupled system is $(T_{jk}) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. Identify which of the following pairs of vectors are T-orthogonal
(a) $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}\right\}$ (b) $\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$
(c) $\left\{\begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ (d) $\left\{\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right\}$

(vii) The components of the strain tensor are given by

(a) $\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ (b) $\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}$

(c) $\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}\right)$ (d) $\frac{\partial u_i}{\partial x_j}-\frac{\partial u_j}{\partial x_i}$

(viii) Using Einstein's summation convention, the condition for mechanical equilibrium of a continuous body under the action of external forces is given by

(a) $\frac{\partial \sigma_{ij}}{\partial x_i}=0$ (b) $\frac{\partial \sigma_{ji}}{\partial x_i}=0$

(c) Both (a) and (b) (d) None of the above.

(ix) A velocity field is given by $\vec{V}=2xy\hat{\imath}-3y^2\hat{\jmath}$. The flow field is

(a) Incompressible and steady (b) Incompressible and unsteady

(c) Compressible and steady (d) Compressible and unsteady.

(x) Which of the following relations implies a nonholonomic constraint?

(a) $\dot{x}\sin x-\dot{y}\cos y=0$ (b) $\dot{x}\,sin^2\,x-\dot{y}\,cos^2\,y=0$

(c) $\dot{x}\sin y-\dot{y}\cos x=0$ (d) $\dot{x}x^2+\dot{y}y^2=0$

Group- B

2. (a) For a system with two particles of mass M and m such that $M>m$, show that centre of mass lies closer to mass M. [(CO1)(Remember/LOCQ)]

(b) Find the centre of mass of a uniform plate bounded by $y=\sin x$ and the x axis, $0\le x\le\pi$. [(CO1)(Remember/LOCQ)]

(c) Find the moment of inertia of a rigid hollow uniform circular cylinder of inner radius R_1 and outer radius R_2 having mass M for rotation about its axis. Explain why the products of inertia are zero. [(CO1) (Understand/LOCQ)]

(d) Construct the Euler's equation for a rotating rigid body. [(CO1)(Apply/IOCQ)]

2 + 3 + (3 + 1) + 3 = 12

3. (a) Determine the relation between moments of inertia about two parallel set of axes (X,Y,Z) and (X′,Y′,Z′) respectively, such that one set of axes passes through the centre of mass (at point O) of the rigid body and the origin of the other set is shifted to point O′, where OO′=$\vec{a}$. [(CO1)(Evaluate/HOCQ)]

(b) Determine the inertia tensor of a uniform square plate of length 'a' about the one of the corners at origin O and x,y and z axes such that x and y axes are parallel to edges of the square plate. Further, evaluate the principal moments and directions of the principal axes for the plate. [(CO1) (Evaluate/HOCQ)]

(c) For a symmetric top with two equal principal moments of inertia, discuss how one of the components of the angular velocity as measured from body frame is constant from Euler's equations with zero torque. [(CO1)(Create/HOCQ)]

3 + (2 + 2 + 3) + 2 = 12

Group - C

4. (a) Define action A corresponding to a Lagrangian $\doteq L(q, \dot{q}, t)$, where, q and $\dot{q} = \frac{dq}{dt}$ are generalized coordinate and generalized velocity respectively with t being the time. [(CO2)(Remember/LOCQ)]

(b) (i) Apply the Euler-Lagrange Equation of motion to show that a Lagrangian $L = M[\dot{x}(\dot{x} + y + z) + \dot{y}(\dot{y} + x + z) + \dot{z}(\dot{z} + x + y)]$ represents the motion of a free-particle of mass M. [(CO3)(Apply/IOCQ)]

(ii) Construct the Hamiltonian for the system. [(CO3)(Create/HOCQ)]

(c) A system has a Lagrangian $L = L(x, \dot{x}, t)$ Show that $J = \frac{\partial L}{\partial \dot{x}}\dot{x} - L$ is a constant of motion if L is not an explicit function of t. [(CO2)(Understand/LOCQ)]

(d) The point of suspension of a pendulum is attached to a horizontal spring vibrating following the equation $x = a \sin \omega t$. Consider that the gravity is acting along positive Y-direction. Construct the constraint relations for the system? [(CO3)(Create/HOCQ)]

2 + (3 + 3) + 3 + 1 = 12

5. (a) The Lagrangian of a systems given by $L(q, \dot{q}) = \frac{1}{2}m\dot{q}_1{}^2 + \frac{1}{2}Mq_1{}^2\dot{q}_2{}^2 + m\dot{q}_3{}^2 - \frac{1}{2}kq_2{}^2$, where $\{q_j: j = 1, 2, 3\}$ are generalized coordinates and $\{m, M, k > 0\}$ are constants. Evaluate the components of generalized momentum. [(CO3)(Evaluate/HOCQ)]

(b) The point of suspension of a pendulum of mass m and string length l is moving with velocity $\vec{v} = 5\hat{\imath} - 10t\hat{\jmath}$ starting from the rest. Construct the constraint relations for the pendulum. [(CO3)(Create/HOCQ)]

(c) A metal chain of linear mass density μ is hanging between two horizontal points in the presence of constant gravity.

(i) Construct the Lagrangian of the problem. [(CO2)(Apply/IOCQ)]

(ii) Show that the shape of the chain is a catenary. [(CO2)(Understand/LOCQ)]

3 + 3 + (2 + 4) = 12

Group - D

6. The kinetic and potential energies of a system are given by$T = \frac{1}{2}m\left(\dot{x}_1{}^2 + \dot{x}_2{}^2\right)$ and $V = \lambda[x_1(e^{-kx_2} - 1) + x_2(e^{-kx_1} - 1) + \frac{1}{2}(x_1{}^2 + x_2{}^2)]$ respectively with mass $m, \lambda > 0$.

(i) Prove that the point $(x_1, x_2) = (0, 0)$ is a point of equilibrium. [(CO4)(Evaluate/HOCQ)]

(ii) Identify the condition on k for which small oscillation is possible. [(CO4)(Apply/IOCQ)]

(iii) Determine the normal mode frequencies. [(CO4)(Evaluate/HOCQ)]

(4 + 4 + 4) = 12

7. A particle m is moving in a potential $V \doteq V(q)$which is differentiable up to any arbitrary order at the $q = 0$.

(i) If the particle performs a simple harmonic motion, show that the Lagrangian is given by $L = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}\frac{d^2V}{dq^2}|_{q=0}q^2$. [(CO4)(Remember/LOCQ)]

(ii) Construct the Lagrange equations of motion. [(CO4)(Apply/IOCQ)]

(iii) Show that the frequency is given by $\omega = \sqrt{\frac{1}{m}\frac{d^2V}{dq^2}|_{q=0}}$.
[(CO4)(Remember/LOCQ)]

(iv) Prove that such a system is conservative. [(CO4)(Evaluate/HOCQ)]

(4 + 2 + 2 + 4) = 12

Group - E

8. (a) A steel bar ($Young's\ modulus Y = 200 \times 10^9\ N/m^2$, $Poisson's\ ratio v = 0.3$) of a circular cross section (diameter $4\ cm$) and $1\ m$ in length, is subjected to an axial tensile force of $250000\ N$. Find the decrease in the lateral dimension due to this load. [(CO6)(Remember/LOCQ)]

(b) Consider a body of arbitrary shape in equilibrium under the action of external deforming forces. Analyse the stress components on the six sides of an elementary parallelepiped inside this body using a suitable diagram.
[(CO6)(Analyse/IOCQ)]

(c) Develop the momentum equation in differential form for an inviscid fluid. Assume that the only body force acting on the fluid is gravity.
[(CO5)(Apply/IOCQ)]

(d) Consider the steady flow of a viscous fluid in a long horizontal duct where $\vec{V} = (ay^2 + bz^2)\hat{\imath}$ where a and b are constants. Evaluate the corresponding pressure gradient. The density of the fluid is ρ and its coefficient of viscosity is μ.
[(CO5) (Evaluate/HOCQ)]

3 + 3 + 3 + 3 = 12

9. (a) A solid circular steel rod $6\ mm$ in diameter and $500\ mm$ long is rigidly fastened to the end of a square brass bar $25\ mm$ on a side and $400\ mm$ long, the geometric axes of the bars lying along the same line. An axial tensile force of $5000\ N$ is applied at each of the extreme ends. Determine the total elongation of the assembly. For steel, $Y = 200 \times 10^9\ N/m^2$ and for brass, $Y = 90 \times 10^9\ N/m^2$.
[(CO6)(Evaluate/HOCQ)]

(b) Two points in the undeformed geometry of a body are originally at $(0,0,1)m$ and $(2,0,-1)m$. Determine the distance between these points after deformation when the displacement field $\vec{u} = [(x^2 + y)\hat{\imath} + (3 + z)\hat{\jmath} + (x^2 + 2y)\hat{k}]\ m$ is imposed on the body? [(CO6) (Evaluate/HOCQ)]

(c) For the Cartesian velocity field $\vec{V} = 2x^2y\hat{\imath} + 3\hat{\jmath} + 4y\hat{k}$

(i) Find $\frac{1}{\rho}\frac{D\rho}{Dt}$. [(CO5)(Remember/LOCQ)]

(ii) Find the rate of change of velocity following a fluid particle.
[(CO5)(Remember/LOCQ)]

(d) Water flows in a pipe of cross-sectional diameter $4\ cm$ at $20\ m/s$. Density of water may be assumed to be $1 g/cc$. The pipe divides into two pipes (also of circular cross-sections), one $2\ cm$ in diameter and the other $3\ cm$ in diameter. If

10 kg/s flows from the 2 cm-diameter pipe, solve the appropriate equation to obtain the flow rate and the mass flux from the 3 cm-diameter pipe.

[(CO5)(Apply/IOCQ)]

3 + 3 + (1.5 + 1.5) + (1.5 + 1.5) = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	34.38%	23.96%	41.66%

Course Outcome (CO):

After the completion of the course students will be able to

1. Understand angular momentum kinetic energy and motion of a rigid body with applications in mechanical systems.
2. Understand calculus of variation as a core principle underlying majority of the physical laws: Newton's laws, Laplace equation (electrostatics and fluid mechanics), wave equation, heat conduction equation, control theory and many other.
3. Appreciate dynamical equations as a consequence of variational extremization of action functional along with the use of Euler-Lagrange equation to understand the behaviour of simple mechanical systems.
4. Appreciate the ubiquity of oscillation physics-from pendulum and spring-mass system to electrical circuit and movement of piston and comprehend the small motion of a system around stable equilibrium through the notion of normal modes—the meaning of eigen value problem in oscillation physics.
5. Elucidate the basic principles of fluid mechanics through the study of mass conservation, momentum balance, and energy conservation applied to fluids in motion.
6. Understand the mechanics of deformable bodies through a study of the concepts of normal and shear stresses and strains, following a review of the principles of statics.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
ME - A	https://classroom.google.com/c/NDA2MDQyMjkzMzk4/a/NDc1MTQzMTgzNjM0/details
ME - B	https://classroom.google.com/c/NDA2MTQ2MDcyMDY4/a/NDc1MTQ1Mjk2NTI1/details
BACKLOG	CLASS ROOM LINK:https://classroom.google.com/c/NDc1MDgwMzg3MjI0?cjc=vxq2ael **Submission Link** https://classroom.google.com/c/NDc1MDgwMzg3MjI0/a/NDc1MDgwNDA4NTU5/details