MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) What is the value of m for which $2x - x^2 + my^2$ is harmonic?
(a) -1 (b) 1 (c) -2 (d) 2.

(ii) In the Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ valid in the region $1 < |z| < 2$, the coefficient of $\frac{1}{z^2}$ is
(a) -2 (b) -1 (c) 0 (d) 1.

(iii) The value of the complex integral $\oint_C \frac{1}{z-2} dz$, where C is the circle $|z| < \frac{1}{2}$ is
(a) $2\pi i$ (b) $4\pi i$ (c) 0 (d) 2.

(iv) If $f(x) = x \sin x, -\pi < x \leq \pi$ be expanded in a Fourier series as $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then $a_1 = ?$
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$.

(v) The Fourier transform of $f(t) = \begin{cases} 1 \ for -1 < t < 1 \\ 0\ , \ otherwise \end{cases}$ is
(a) $\sin s$ (b) $\frac{\sin s}{s}$ (c) $\frac{\cos s}{s}$ (d) $\frac{2 \sin s}{s}$.

(vi) The value of $P_n(1)$ is
(a) -1 (b) 1 (c) 0 (d) 2.

(vii) The value of $J_0(0)$ is
(a) 0 (b) -1 (c) 1 (d) $\frac{1}{2}$.

(viii) For $n \geq 1, \int_{-1}^{1} P_n(x)dx =$

(a) -1 (b) 1 (c) 0 (d) 2.

(ix) The solution of $\frac{\partial^2 z}{\partial x^2} = 0$ is (where z is a function of x and y)

(a) $f_1(y) + xf_2(y)$ (b) $(1+x)f(y)$

(c) $x + x^2 f(y)$ (d) $(1+y)f(x)$.

(x) The solution of $q = e^{-p/a}$, where $q = \frac{\partial z}{\partial y},\ p = \frac{\partial z}{\partial x}$, is

(a) $z = \alpha x^2 + \beta y^2$ (b) $z = \alpha x + e^{-\alpha/a} y + \gamma$

(c) $z = (\alpha + \beta)x^2$ (d) $z = (\alpha + \beta)y^2$.

where α, β, γ are constants.

Group–B

2. (a) (i) Prove that $u(x,y) = e^{-x}(x \sin y - y \cos y)$ is harmonic.

(ii) Use the Cauchy-Riemann equations to find $v(x,y)$ such that $f(z) = u + iv$ is analytic. [(CO2) (Remember/LOCQ)]

(b) Evaluate $\oint_C \frac{z^2+1}{z^2-1} dz$, where C is the circle (i) $|z| = \frac{3}{2}$, (ii) $|z| = \frac{1}{2}$. [(CO2) (Evaluate/HOCQ)]

(2 + 4) + (3 + 3) = 12

3. (a) Expand $f(z) = \frac{z}{(z+1)(z+3)}$ in a Laurent series valid for (i) $1 < |z| < 3$, (ii) $|z| < 1$. [(CO2) (Understand/LOCQ)]

(b) Evaluate the following integral using the Residue Theorem.

$\oint_C \frac{4-3z}{z(z-1)(z-2)} dz$, where C is the circle $|z| = \frac{3}{2}$. [(CO2) (Evaluate/HOCQ)]

(3 + 3) + 6 = 12

Group - C

4. (a) Use Parseval's Identity to prove that $\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab\,(a+b)}$, $(a > 0, b > 0)$.

Hence show that $\int_0^\infty \frac{dt}{(1+t^2)^2} = \frac{\pi}{4}$. [(CO3, CO4) (Apply/IOCQ)]

(b) Find the Fourier series of the function $f(x) = x^2$ in $(-\pi, \pi)$ and hence show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$. [(CO3, CO4)(Apply/IOCQ)]

6 + 6 = 12

5. (a) Find the half-range Fourier cosine series for the function $f(x) = \pi - x$ in $0 < x < \pi$. [(CO3, CO4)(Remember/LOCQ)]

(b) (i) Evaluate $F^{-1}\left\{\frac{1}{s^2+4s+13}\right\}$

(ii) Find the Fourier cosine transform of $f(x) = \begin{cases} x & for\ 0 < x < 1, \\ 2 - x & for\ 1 < x < 2 \\ 0 & for\ x > 2. \end{cases}$

[(CO3, CO4) (Apply/IOCQ)]

6 + (3 + 3) = 12

Group - D

6. (a) Find the solution in series of $y'' + xy' + x^2y = 0$ about $x = 0$.
[(CO5) (Evaluate/HOCQ)]

(b) Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. [(CO5)(Apply/IOCQ)]

7 + 5 = 12

7. (a) Prove that $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$. [(CO5) (Analyse/IOCQ)]

(b) Show that $\sin x = 2J_1 - 2J_3 + 2J_5 - \cdots$ using the generating function of the Bessel functions. [(CO5)(Evaluate/HOCQ)]

6 + 6 = 12

Group – E

8. (a) Form a PDE by eliminating the function φ from $\varphi\left(\frac{z}{x^3}, \frac{y}{x}\right) = 0$.
[(CO6) (Create/HOCQ)]

(b) Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = x^2y^2$. [(CO6) (Apply/IOCQ)]

7 + 5 = 12

9. (a) Solve $\left(2D^2 - 5DD' + 2D'^2\right) z = 24(y - x)$, where $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial y}$.
[(CO6)(Understand/LOCQ)]

(b) Find the solution of the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, using the method of separation of variables. (Here $u(x,t)$ is the temperature of a bar at any time t, at a distance x from the origin). [(CO6) (Understand/LOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	25%	42.70%	32.30%

Course Outcome (CO):

After the completion of the course students will be able to

MATH2001.1 Construct appropriate mathematical models of physical systems.

MATH2001.2 Recognize the concepts of complex integration, Poles and Residuals in the stability analysis of engineering problems.

MATH2001.3 Generate the complex exponential Fourier series of a function and make out how the complex Fourier coefficients are related to the Fourier cosine and sine coefficients.

MATH2001.4 Interpret the nature of a physical phenomena when the domain is shifted by Fourier Transform e.g. continuous time signals and systems.

MATH2001.5 Develop computational understanding of second order differential equations with analytic coefficients along with Bessel and Legendre differential equations with their corresponding recurrence relations.

MATH2001.6 Master how partial differentials equations can serve as models for physical processes such as vibrations, heat transfer etc.

Department & Section	Submission Link (Regular)
ECE-A	https://classroom.google.com/c/NDA2NTU2NTYxNDYw/a/NDY0MTk3Nzg4ODUx/details
ECE-B	https://classroom.google.com/c/NDA5ODM0NTI0OTgz/a/NDY0MTk3Nzg5MTkx/details
ECE-C	https://classroom.google.com/c/NDA2MTMxMzgwMDc0/a/NDY0MTk4NzA4NjE4/details
AEIE	https://classroom.google.com/c/NDA2MTE2MTMwMjg5/a/NDY0MjY3ODYyNTQy/details
ME-A	https://classroom.google.com/c/NDA1MzQ3MjQzMDg5/a/NDY0MzUzMDY5Njc2/details
ME-B	https://classroom.google.com/c/NDA1MzM3MjAzNjIw/a/NDY0MzU3MjU3MTk5/details

Note: Students having backlog in MATH2001 (new syllabus) are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):
https://classroom.google.com/c/NDY0NTA4MTU3MjY4?cjc=yslwkq2

Step-II: Submit your answer script by clicking link below:
https://classroom.google.com/c/NDY0NTA4MTU3MjY4/a/NDY0NTA4MTU3MzQ5/details

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question