MCA/3RD SEM/MATH 2102/2021

INTRODUCTION TO OPTIMIZATION
(MATH 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A feasible solution of LPP
- (a) must satisfy all the constraints simultaneously
- (b) need not satisfy all the constraints, only some of them
- (c) must be a corner point of the feasible region
- (d) all of the above.

(ii) The value of objective function is maximum under linear constraints
- (a) at the centre of feasible region
- (b) at (0,0)
- (c) at any vertex of feasible region
- (d) the vertex which is at maximum distance from (0, 0).

(iii) A basic solution is called non-degenerate, if
- (a) all the basic variables are zero
- (b) none of the basic variables is zero
- (c) at least one of the basic variables is zero
- (d) none of these.

(iv) The initial solution to a transportation problem can be generated in any manner, so long as
- (a) it minimizes cost
- (b) it ignores cost
- (c) all supply and demand are satisfied
- (d) degeneracy does not exist.

(v) The branch-and-bound method is used to solve
- (a) LP problems
- (b) NLP problems
- (c) GP problems
- (d) IP problems.

(vi) In a pure strategy game
- (a) any strategy may be selected arbitrarily
- (b) a particular strategy is selected by each player

(c) both players select their optimal strategy
(d) none of these.

(vii) If the losses of player A are the gains of the player B, then the game is known as
(a) Fair game (b) Unfair game
(c) Non-zero-sum game (d) Zero-sum game.

(viii) When the game is not having a saddle point, then the following method is used to solve the game:
(a) Linear Programming method (b) Minimax and maximin criteria
(c) Algebraic method (d) Graphical method.

(ix) The critical path
(a) Is a path that operates from the starting node to the end node
(b) Is a mixture of all paths
(c) Is the longest path
(d) Is the shortest path.

(x) Activity in a network diagram is represented by?
(a) Arrows (b) Circles (c) Squares (d) Rectangles.

Group – B

2. (a) Given the LPP
Maximize $Z = 2x_1 + 3x_2 + 4x_3$
subject to the constraints
$x_1 - 5x_2 + 3x_3 = 7,$
$2x_1 - 5x_2 \leq 3$
$3x_2 - x_3 \geq 5$
$x_1, x_2 \geq 0,$ and
x_3 is unrestricted in sign.
Formulate the dual of the LPP. [(CO1) (Create/HOCQ)]

(b) Solve the following LPP problem graphically:
Maximize $Z = -x_1 + 2x_2$
subject to the constraints
$x_1 - x_2 \leq -1,$
$-0.5x_1 + x_2 \leq 2,$
and $x_1, x_2 \geq 0$. [(CO1) (Apply/IOCQ)]

6 + 6 = 12

3. (a) A company makes two kinds of leather belts, belt A and belt B. Belt A is a high-quality belt and belt B is of lower quality. The respective profits are Rs 4 and Rs 3 per belt. The production of each of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1,000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle and only 400 of these are available per day. There are only 700 buckles a day available for belt B.

What should be the daily production of each type of belt? Formulate this problem as an LP model and convert it to its standard form. [(CO1)(Create/HOCQ)]

(b) Solve the following LPP using Simplex method:
Maximize $Z = 10x_1 + 12x_2 + 12x_3$
subject to the constraints
$x_1 + 2x_2 + 2x_3 \leq 20$
$2x_1 + x_2 + 2x_3 \leq 20$
$2x_1 + 2x_2 + x_3 \leq 20$
$x_1, x_2, x_3 \geq 0$ [(CO1) (Apply/IOCQ)]

6 + 6 = 12

Group – C

4. A company has three production facilities S1, S2 and S3 with production capacity of 7, 9 and 18 units (in 100s) per week of a product, respectively. These units are to be shipped to four warehouses D1, D2, D3 and D4 with requirement of 5, 6, 7 and 14 units (in 100s) per week, respectively. The transportation costs (in rupees) per unit between factories to warehouses are given in the table below:

	D1	D2	D3	D4	Supply
S1	19	30	50	10	7
S2	70	30	40	60	9
S3	40	8	70	20	18
Availability	5	8	7	14	34

(i) Formulate the transportation problem as an LP model to minimize the total transportation cost. [(CO2) (Create/HOCQ)]
(ii) Find an initial basic feasible solution to the transportation problem using Vogel's Approximation Method (VAM). [(CO2) (Apply/IOCQ)]

(6 + 6) = 12

5. (a) A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.

		Employees				
Jobs		I	II	III	IV	V
	A	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

How should the jobs be allocated, one per employee, so as to minimize the total man-hours? [(CO2) (Understand/LOCQ)]

(b) Solve the following integer programming problem using branch and bound method.
Maximize $Z = 5x_1 + 7x_2$
subject to the constrains
$2x_1 + x_2 \leq 13,$

$5x_1 + 9x_2 \leq 41,$
$x_1, x_2 \geq 0$ are integers. [(CO4) (Apply/IOCQ)]

6 + 6 = 12

Group – D

6. (a) Find the range of values of p and q which will render the entry (2,2) a saddle point for the game. [(CO3)(Understand/LOCQ)]

		Player B		
Player A		$B1$	$B2$	$B3$
	$A1$	2	4	5
	$A2$	10	7	q
	$A3$	4	p	6

(b) Two player A and B match coins. If the coins match, then A wins two units of value, if the coin does not match, then B win 2 units of value. Determine the optimum strategies for the players and the value of the game. [(CO3)(Evaluate/HOCQ)]

6 + 6 = 12

7. (a) For the game with payoff matrix

	Player B		
Player A	−1	2	−1
	6	4	−6

Determine the best strategies for players A and B and also the value of the game. Is this game(i) fair (ii) strictly determinable? [(CO3)(Understand/LOCQ)]

(b) Solve the following game using graphical method. [(CO3) (Apply/IOCQ)]

	Player B		
Player A	4	−1	0
	−1	4	2

6 + 6 = 12

Group – E

8. (a) Construct an activity on arrow network based on the activity descriptions below. Show all your work. Label activities in the network by their activity letters and node numbers. Remove any redundant dependencies and label dummy activities DUMMY1, DUMMY2, etc.

- Activities H, R2, T1 start the project.
- Activity T2 can start when Activities H, E1 and S are completed.
- Activity E1 also depends on Activity R2.
- Activity X follows Activity H and precedes Activity L.
- Activity E is preceded by Activities T2 and P1.
- The predecessors to Activity G are Activities L, T2 and P1.
- The successors to Activity T1 are Activities E1, S, W and D2.
- Activity P1 cannot begin until Activity W is finished.
- Activity P2 and F follow Activities W and D2, and precede Activities E and R1.
- Activity O2 depends on T2 and P1, and precedes Activity L.

[(CO5) (Create/HOCQ)]

(b) A book binder company has one printing machine and one binding machine. There are manuscripts of a number of different books. Processing times for printing and binding are given in the following table:

Book	Time (n hours)	
	Printing	Binding
A	5	2
B	1	6
C	9	7
D	3	8
E	10	4

Determine the sequence in which books should be processed on the machines so that the total time required is minimized. [(CO5) (Analyze/IOCQ)]

6 + 6 = 12

9. (a) The activities associated with a work are summarized in the following table:

No.	Activity	Duration	Predecessors
5	B	5	
10	M	4	B
15	N	9	B
20	Q	15	B
25	A	1	M, N
30	F	4	N, Q
35	X	9	Q
40	C	9	Q
45	Y	9	A, F, X
50	S	6	F
55	J	5	X, F
60	T	10	C
65	V	5	Y, S
70	U	10	V, T, J

(i) Construct a precedence diagram.

(ii) On the diagram, compute the four schedule dates (ESD, EFD, LSD, LFD) and the four floats (TF, FF, INTF, and IDF) for each activity, and the lag for each link. [(CO6) (Apply/IOCQ)]

(b) Find the sequence that minimizes the total time required in performing the following jobs on three machines in order ABC. Processing times (in hours) are given in the following table: [(CO6) (Analyze/IOCQ)]

Job	1	2	3	4	5
Machine A	8	10	6	7	11
Machine B	5	6	2	3	4
Machine C	4	9	8	6	5

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	18.75%	50.00%	31.25%

Course Outcome (CO):

After the completion of the course students will be able to

MATH2102.1 Represent real-world optimization problems by mathematical models and solve them by various techniques.

MATH2102.2 Categorize Transportation and Assignment problems.

MATH2102.3Apply the way in which Game Theoretic Models can be made useful to a variety of real-world scenarios in economics and in other areas.

MATH2102.4. Understand the limitations of simplex method and have realistic approach towards practical problems using Integer Linear Programming Problem.

MATH2102.5. Understand the significance of using PERT and CPM techniques for project management.

MATH2102.6. Solve some specific problems for scheduling jobs on machines.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
MCA	https://classroom.google.com/c/NDAwNTk0MDg2MjA5/a/NDYzOTUyNTI0OTg5/details