DISCRETE MATHEMATICS
(MCAP 1104)

Time Allotted : 3 hrs   **Full Marks : 70**

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

    (i)   If the general term of the sequence $\{ a_k \} = a^k$,then the corresponding generating function is
            (a) $1/(1-ax)$ (b) $1/(1-x)$
            (c) $k/(1-x)$ (d) $a/(1-x)$

    (ii)   If n be the number of vertices, e be the number of edges and k be the number of components then
            (a) $e > n+k$ (b) $e \geq n-k$
            (c) $e \leq n-k$ (d) none of these.

    (iii)   If the function $f : R \rightarrow R$ is defined by $f(x) = 3x-4$, when $x>0$
                                        $= -3x+2$, when $x \leq 0$
            then $f^{-1}(2) =$
            (a) $\{2\}$ (b) $\{0,2\}$
            (c) $\{-2,2\}$ (d) none of these

    (iv)   The cardinality of a power set of a non-empty set A is
            (a) $2^{|A|}$ (b) $2|A|$
            (c) $|A|^2$ (d) $|A|^2 - |A|$.

    (v)   How many ways can the letters of the word "LEATHER" be arranged
            (a) 72   (b)144   (c) 360   (d) none of these.

    (vi)   The proposition $P \wedge (\sim P \vee Q)$ is a
            (a) Tautology   (b) logically equivalent to $P \wedge Q$
            (c) logically equivalent to $P \vee Q$ (d) a contradiction.

    (vii)   What is the minimum number of vertices necessary for a graph with 6 edges
            (a) 6   (b) 5   (c) 7   (d) none of these.

(viii)   Which of the following is not true?
(a) $A \oplus B = (A-B) \cap (B-A)$ (b) $A \cup A^c = U$
(c) $A^c \cup B^c = (A \cap B)^c$ (d) $A-B = B^c - A^c$

(ix)   A spanning tree has
(a) only one circuit   (b) two circuit
(c) no circuit   (d) none of these.

(x)   Find the rank of the word LETTER, when the letters are arranged as in dictionary.
(a) 13   (b) 14   (c) 15   (d) 16.

Group- B

2.   (a)   Prove that for any three sets A,B,C : $A \times (B \cup C) = (A \times B) \cup (A \times C)$
[(CO1) (Remember/LOCQ)]
   (b)   A relation ρ is defined on the set N of natural numbers such that "m ρ n iff m is a divisor of n" for all   m,n $\in$ N .Examine if ρ is
(i) reflexive (ii) symmetric (iii) transitive.   [(CO1) (Understand/LOCQ)]
   (c)   Prove that $(P \wedge (P \leftrightarrow Q)) \rightarrow Q$ is a tautology.   [(CO1)(Analyze/IOCQ)]
3 + 6 + 3 = 12

3.   (a)   Prove that for n variables the sum of all minterms =1.
[(CO1) (Remember/LOCQ)]
   (b)   Let f: R$\rightarrow$ R be defined by f(x) = 3x+1; for all x$\in$ R. [(CO1) (Understand/LOCQ)]
Examine if f is (i) injective (ii) surjective.
   (c)   Let A ={a , b} and P(A) be it's power set. Let $\subseteq$ be the inclusion relation on the elements of (P(A), $\subseteq$). Draw the Hasse diagram for (P(A), $\subseteq$).
[(CO1)(Analyse/IOCQ)]
3 + 6 + 3 = 12

Group - C

4.   (a)   Find the coefficient of x^{11} in $(1+x+x^2+\ldots\ldots+x^5)^4$ [(CO2) (Remember/LOCQ)]
   (b)   Find the integer solution for the Diophantine equation 65x+14y=4.
[(CO2)(Analyze/LOCQ)]
   (c)   Solve the recurrence relation by generating function [(CO2)(Analyze/IOCQ)]
$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for n$\geq$2. Where a_0=10, a_1=41.
3 + 5 + 4 = 12

5.   (a)   What is the number of permutations in the letters of the word MISSISSIPPI where 4S's don't come   together. [(CO2)(Understand/LOCQ)]
   (b)   Show that if any 30 people are selected, then we can choose a subset of 5 so that all 5 were born on the same day of the week. [(CO2)(Understand/LOCQ)]
   (c)   Consider a set of integers from 1 to 250.Find how many of these numbers are divisible by 3 or 7 or 5. Also indicate how many are divisible by 3 or 7 but not by 5 and divisible by 3 or 5. [(CO2)(Analyze/IOCQ)]
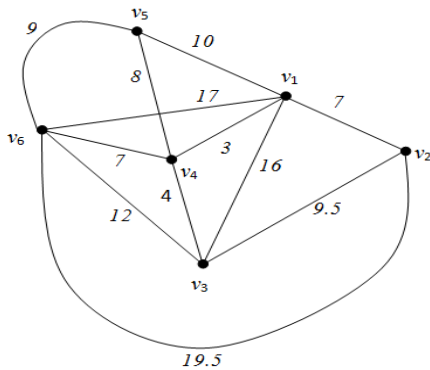**3 + 6 + 3 = 12**

Group - D

6. (a) Prove that the number of vertices of odd-degree in a graph G is always even.
        [(CO3) (Remember/LOCQ)]

    (b) Prove that the maximum number of edges in a graph G with n vertices and k components is
        (n-k)(n-k+1)/2   [(CO3) (Remember/LOCQ)]

    (c) Write short notes on the following (any two)
        (i) Walk   (ii) Hamiltonian path.   (iii) Spanning Tree.
        [(CO3)(Analyze/IOCQ)]

                                                          3 + 6 + 3 = 12

7. (a)

        Apply Prim's algorithm to draw the minimum spanning tree for the above connected graph and also find the length of the minimum spanning tree. [(CO3) (Remember/LOCQ)]

    (b) Prove that a tree with n vertices always has (n-1) edges.
        [(CO3)(Understand/LOCQ)]

                                                          6 + 6 = 12

Group - E

8. (a) Briefly discuss with example type-0,type-1,type-2,type-3 grammars.
        [(CO4) (Remember/LOCQ)]

    (b) From the state transition table of the following NFA obtain it's equivalent DFA.

S \ Σ	f O	1
S_0	$\{S_0,S_1\}$	$\{S_1\}$
S_1	$\{S_2\}$	$\{S_2\}$
S_2	f	$\{S_2\}$

                                                          6 + 6 = 12

9. (a) Convert to a Mealy-machine which is equivalent to the given Moore-machine given in the following table.

Present state	Next state		Output
	a=0	a=1	
Q_0	Q_3	Q_1	1

Q_1	Q_1	Q_2	0
Q_2	Q_2	Q_3	0
Q_3	Q_3	Q_0	0

[(CO4)(Understand/HOCQ)]

(b) Construct a Moore-machine equivalent to the Mealy-machine M given by the following table. [(CO4)(Analyse/HOCQ)]

Present State	Next State			
	a=0		a=1	
	State	Output	State	Output
→Q	Q_1	1	Q_2	0
Q	Q_4	1	Q_4	1
Q	Q_2	1	Q_3	1
Q	Q_3	0	Q_1	1

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	30%	43.33%	26.67%

Course Outcome (CO):

After the completion of the course students will be able to

(1) Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of Set, Relations, Mapping, Propositional Logics etc.
(2) Interpret the problems that can be formulated in terms of generating functions and permutations. Apply counting techniques and the crucial concept of recurrence.
(3) Interpret the problems that can be formulated in terms of graphs and trees.
(4) Analyze the logical fundamentals of basic computational concepts.
(5) Compare the notions of converse, contrapositive, inverse etc. in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.
(6) Achieve the ability to think and reason abstract mathematical definitions and ideas.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
MCA	https://classroom.google.com/c/NDczMzczNzUxMTI3/a/NDc1MDQ4ODMxODg4/details