MATHEMATICAL & STATISTICAL METHODS
(MATH 2101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In Trapezoidal rule for finding the value of $\int_a^b f(x)dx$ there exist no error if $f(x)$ is
(a) parabolic function (b) linear function
(c) logarithmic function (d) exponential function.

(ii) The value of a_0 in the Fourier series of $f(x) = x^3 cos2x$ in $(-2\pi, 2\pi)$ is
(a) 0 (b) 1 (c) 2π (d) $8\pi^3$.

(iii) Find $f(10)$ from the following data:

x	6	9	12
$f(x)$	24	36	48

(a) 38 (b) 42 (c) 40 (d) 44.

(iv) The Fourier series of a function exist if the function is periodic and it satisfies
(a) Lagrange's conditions. (b) Dirichlet's conditions.
(c) Legendre's conditions. (d) Cauchy's conditions.

(v) If the discrete random variables X has the distribution function $F(X)$ and $F(0) = \frac{1}{6}, F(1) = \frac{1}{2}, F(3) = \frac{3}{4}$, the probability $P(0 < X \leq 3)$ is
(a) $\frac{3}{4}$ (b) $\frac{1}{3}$ (c) $\frac{7}{12}$ (d) $\frac{1}{12}$.

(vi) The period of the function $f(x) = 3sinx + cos2x$ is
(a) 0 (b) $\frac{\pi}{2}$ (c) π (d) 2π.

(vii) If X has exponential distribution with parameter $\lambda = 1$ then $P(X > 3) =$
(a) $\frac{1}{e^6}$ (b) e^3 (c) e (d) $\frac{1}{e^3}$.

(viii) The complete solution of the equation $z = px + qy + 2pq$ is
(a) $z = ax + by + 2ab^2$. (b)$z = ax - by - 2ab$.

(c) $z = ax + by + ab$. (d) $z = ax + by + 2ab$.

where a, b are constants and $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

(ix) The standard deviation of the following observation $5, 7, 1, 2, 6, 3$ is
(a) 4.66 (b) 2.16 (c) 1.47 (d) 0.

(x) The order and degree of the following partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = xy$ is
(a) $2, 0$ (b) $2, 1$ (c) $1, 1$ (d) $1, 2$.

Group–B

2. (a) Find the partial differential equation by eliminating the arbitrary function φ from the relation $\varphi\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. [(CO5) (Evaluate/HOCQ)]

(b) Solve $z^2(p^2 + q^2 + 1) = c^2$, where $p \equiv \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ using Charpit's method. [(CO5) (Apply/IOCQ)]

6 + 6 = 12

3. (a) Applying Lagrange's method find the solution of the following equation:
$y^2 p - xyq = x(z - 2y)$ where $p \equiv \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ [(CO5)(Apply/IOCQ)]

(b) Solve $\left(D^2 - 3DD' + 2D'^2\right)z = 2e^{2x-y} + 4\cos(x + 2y)$ where $D \equiv \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$. [(CO5) (Understand/LOCQ)]

6 + 6 = 12

Group - C

4. (a) Estimate the population(in million) of a city in the year 1955 from the data given in the following table:

Year	1951	1961	1971	1981
Population (in million)	35	42	58	84

[(CO1, CO2) (Apply/IOCQ)]

(b) A function $f(x)$ is given as a table of values. Approximate the area under the curve $y = f(x)$ from $x = 1$ to $x = 3$ using Simpson's $\frac{1}{3}$rd rule.

x	1	1.5	2	2.5	3
$f(x)$	5.88	8.76	12.47	18.5	25.78

[(CO1, CO2) (Apply/IOCQ)]

6 + 6 = 12

5. (a) Using suitable interpolation formula find the value of $f(2)$ from the following data.

x	1	3	6	8
$f(x)$	2.5	6.5	8.5	12.5

[(CO1,CO2) (Apply/IOCQ)]

(b) Estimate the value of $\int_0^1 \frac{1}{1+x} dx$ using Trapezoidal Rule taking $h = 0.25$.

[(CO1,CO2) (Apply/IOCQ)]

6 + 6 = 12

Group - D

6. (a) Find the Fourier series of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2, & 1 < x < 2 \end{cases}$. Hence obtain the value of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$. [(CO4) (Understand/LOCQ)]

   (b) Find the half-range Fourier sine series of $g(x) = (x-1)^2$ for $0 < x < 1$.
   [(CO4) (Remember/LOCQ)]

7 + 5 = 12

7. (a) Let $f(x)$ be a square wave function defined by $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$. Find the Fourier series of this function $f(x)$.
   [(CO4) (Understand/LOCQ)]

   (b) Express $f(x) = x$ in half range Fourier cosine series in $(0, 1)$ and hence obtain the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \cdots \ldots \ldots \ldots$ by applying the Perseval's Identity.
   [(CO4)(Apply/IOCQ)]

5 + 7 = 12

Group – E

8. (a) The number of emergency calls received during a ten minute interval on a hospital switch board is found to have a Poisson distribution with mean 5. Find the probability that there are (i) at most 2 emergency calls in a ten minute interval and (ii) exactly 3 emergency calls in a ten minute interval.
   [(CO3, CO6)(Remember/LOCQ)]

   (b) 100 unbiased coins are tossed. Using normal approximation to binomial distribution, calculate the probability of getting (i) exactly 40 heads and (ii) 55 heads or more.
   Given, $\Phi(2.1) = 0.9821, \Phi(1.9) = 0.9713, \Phi(0.9) = 0.8159$.
   [(CO3, CO6) (Apply/IOCQ)]

6 + 6 = 12

9. (a) The arithmetic mean calculated from the following frequency distribution is known to be 67.45 inches. Find the value of f_3.

$Height(in\ inches)$:	$60-62$	$63-65$	$66-68$	$69-71$	$72-74$
Frequency:	15	54	f_3	81	24

   [(CO3,CO6) (Analyse/IOCQ)]

   (b) Calculate the correlation co-efficient between x and y for the following data:

x	1	2	3	4	5	6	7	8	9
y	9	8	7	6	5	4	3	2	1

   Also find the regression line of x on y. [(CO3,CO6) (Evaluate/HOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	30.21%	57.29%	12.5%

Course Outcome (CO):

After the completion of the course students will be able to

1. Apply numerical methods to obtain approximate solutions to mathematical problems where analytic solutions are not possible.
2 Implement appropriate numerical methods for solving advanced engineering problems dealing with interpolation and integration.
3. Design stochastic models to predict the outcomes of events.
4. Recognize the significance of the expansion of a function in Fourier Series.
5. Provide deterministic mathematical solutions to physical problems through partial differential equations.
6. Employ statistical methods to make inferences on results obtained from an experiment.

Department & Section	Submission Link (Regular)
BT	https://classroom.google.com/c/NDI1NTIyMTExNDE2/a/NDY0MDA5NzQ2MTU1/details

Note: Students having backlog in MATH2101 are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):
https://classroom.google.com/c/NDY0ODY1ODAwMjcx?cjc=hojyrn6

Step-II: Submit your answer script by clicking link below:
https://classroom.google.com/c/NDY0ODY1ODAwMjcx/a/NDY0ODY1OTA4Nzkx/details