NUMERICAL METHODS OF ANALYSIS
(CHEN 3104)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The error comes because of representing 0.2011456 as 0.2011 is called _____
(a) rounding error (b) truncation error
(c) chopping error (d) approximate error

(ii) Heun's Method of ODE integration _____
(a) is similar to 2nd order Runge Kutta
(b) is similar to 1st order Runge Kutta
(c) is modified Euler method
(d) is a separate method from the methods said in (a), (b) and (c)

(iii) Simpson's 1/3 rule is used to evaluate _____
(a) numerical integration
(b) numerical differentiation
(c) solution of nonlinear equation
(d) solution of linear simultaneous equation

(iv) Given the two points [a,f(a)], [b,f(b)], the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by _____
(a) $f_1(x)=\frac{x+b}{b-a}f(a)+\frac{x+a}{b-a}f(b)$
(b) $f_1(x)=\frac{x-b}{b-a}f(a)+\frac{x-a}{b-a}f(b)$
(c) $f_1(x)=\frac{x+b}{b-a}f(b)+\frac{x+a}{b-a}f(a)$
(d) $f_1(x)=\frac{x-b}{b-a}f(b)+\frac{x-a}{b-a}f(a)$

(v) Gauss Seidal method is a method of _____
(a) iterations (b) false positions
(c) successive displacement (d) elimination

(vi) For the given distributed data the value of $\Delta^3 y_0$ is _____

x	3.6	3.65	3.7	3.75
y	36.6	38.5	40.4	42.5

(a) 0.095 (b) 0.007 (c) 1.872 (d) 0.123

(vii) The convergence of which of the following method is sensitive to starting value?
(a) false position method (b) Gauss Siedel method
(c) Newton-Raphson method (d) all of these

(viii) Unsteady heat conduction in a thin circular rod, the ends of which are kept at two different temperatures give rise to _____
(a) initial value problems (b) partial differential equations
(c) boundary value problem (d) first order ODE

(ix) An algorithm is said to be stable if _____
(a) Results change slightly with perturbations in the independent variable
(b) Results change grossly with perturbations in the dependent variable
(c) Results do not change with any perturbation
(d) Results decay over time in time-dependent problem

(x) In the generic PDE given by $A\frac{\partial^2 T}{\partial x^2}+B\frac{\partial^2 T}{\partial y \partial x}+C\frac{\partial^2 T}{\partial y^2}+D=0$ the equation is parabolic if B^2-4AC is _____
(a) = 0 (b) > 0 (c) < 0 (d) =1

Group - B

2. (a) What are rounding and chopping error? Why are they appearing during the application of numerical algorithm? [(CO5) (Remember/LOCQ)]

(b) Explain how you assess the convergence of Newton-Raphson algorithm using condition number? [(CO3) (Evaluate/HOCQ)]

(c) A very simplified model of a fuel cell reaction suggests a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, T(s) is given by:

$$T(s)=\int_{C_o}^{C_f}\frac{6.73c+4.3025x10^{-7}}{2.316x10^{-09}c}dc$$

Anode: $2H_2 \rightarrow 4H^+ + 4e^-$; Cathode: $O_2 + 4H^+ + 4e^- \rightarrow 2H_2O$
If the initial concentration of oxygen is $1.22x10^{-6}$ g/cc, find the time required for the concentration to drop to 50% of its initial value. Use Simpson 1/3 rule.
[(CO4) (Apply/IOCQ)]

3 + 3 + 6 = 12

3. (a) Water solubility in jet fuel , W_S, as a function of temperature, T, has been measured and the following experimental data reported.

T (°C)	-40	-20	0	20	40
W_S	0.0012	0.002	0.0032	0.006	0.0118

Use Newton's interpolation to obtain values at 5 °C. [(CO4) (Apply/IOCQ)]

(b) If a Lagrange polynomial were to be fitted to above data, evaluate the coefficients and state the order of polynomial used. [(CO3) (Evaluate/HOCQ)]

6 + 6 = 12

Group - C

4. (a) You have a spherical storage tank containing oil (Fig. 1). The tank has a diameter of 6 m. You are asked to calculate the height 'h' to which a dipstick 8 m long would be wet with oil when immersed in the tank when it contains 10 m^3 of oil.

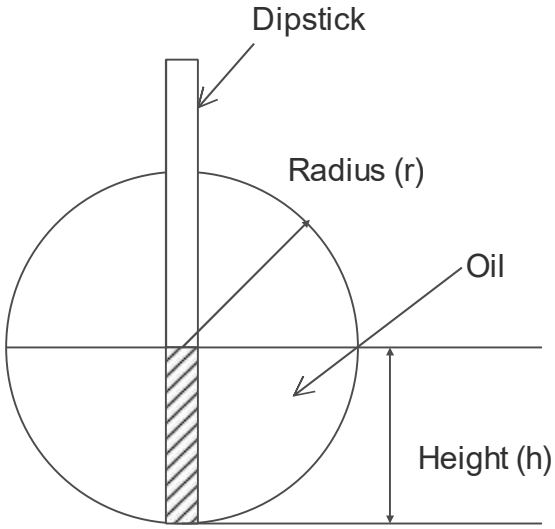

Fig. 1

The equation that gives the height (h) of the liquid in the spherical tank for the given volume and radius is given by $f(h) = h^3 - 9h^2 + 3.8197 = 0$

Use the bisection method of finding roots of equations to find the height (h) to which the dipstick is wet with oil. Show maximum three iterations if not converged. [(CO2) (Apply/IOCQ)]

(b) Which method will provide faster convergence – "secant method" or "bisection method"? Explain with proper reasoning. [(CO3) (Evaluate/HOCQ)]

9 + 3 = 12

5. (a) The following system of equations is designed to determine concentrations (the c's in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day),

$$15c_1 - 3c_2 - c_3 = 3300$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2400$$

Solve the system using LU decomposition method. [(CO4) (Apply/IOCQ)]

(b) For the previous problem of 4(a), determine how much the rate of mass input to reactor 3 must be increased to induce a 10 g/m^3 rise in the concentration of reactor 1 in the above problem. [(CO4) (Apply/IOCQ)]

10 + 2 = 12

Group - D

6. (a) Write down the algorithm for 4th order Runge-Kutta method. [(CO2)(Remember/LOCQ)]

   (b) Show that in order to gain the accuracy in the predicted solution for ODE using Heun's method, the step size must be considered infinitesimally small. [(CO6) (Create/HOCQ)]

   (c) A chemical compound decays over time, when exposed to air and can be modelled using the equation given below:

$$\frac{dn}{dt} = -0.8n^{1.5} + 7ne^{-3t}$$

   Solve the equation using Heun's method to find the concentration 'n' as a function of time using a time step of 0.5 s (show two iterations). n(0)= 4000. [(CO1) (Apply/IOCQ)]

3 + 4 + 5 = 12

7. (a) A non-isothermal batch reactor has a nonlinear model. C, concentration, and T (in K) the temperature in the reactor can be modelled using the following system of equations

$$\frac{dC}{dt} = -Ce^{(-10/T)}$$

$$\frac{dT}{dt} = -10(T-20) + T/C$$

   Initially the reactor is at 310°K and concentration is 1gmol/L. Find the temperature, *T* and concentration, *C* after 0.03 s. [(CO1) (Apply/IOCQ)]

   (b) How would the equations look if this were an isothermal batch process with temperature kept at T_0?. If $T_0=100K$, analytically integrate the system. [(CO2) (Apply/IOCQ)]

6 + 6 = 12

Group - E

8. (a) Write down 2D PDEs for Laplace equation, Poisson equation and Parabolic equation. [(CO2) (Remember/LOCQ)]

   (b) The dimensionless temperature distribution in a long thin rod of length 10 cm is governed by the equation:

$$\alpha \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{dt}$$

   The thermal diffusivity, α = 0.835 cm^2/s. The rod is divided into nodes equally spaced at 2 cm interval for finite differencing. The left end of the rod is kept at 80°C and the other end at 20°C. With a time step size of 0.1 s, calculate the temperature at 0.1 s for all intermediate points starting out at T=0. [(CO4) (Apply/IOCQ)]

3 + 9 = 12

9. (a) The following differential equation is to be solved over a domain, 0<x<0.5 and 0<y<0.8 subject to boundary conditions given below.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 y}{\partial y^2}=0.5x$$

$$x=0, \text{for all } y, u=1$$

$$x=0.5, \text{for all } y, \frac{du}{dx}=u$$

$$y=0, \text{for all } x, u=0$$

$$y=0.8, \text{for all } x, u=0$$

Write out the numerical form of the difference equation with the domain divided into 3 interior grid points in the x and y direction. [(CO2) (Apply/IOCQ)]

(b) Setup the system in the form $\underline{\underline{A}}\underline{y}=\underline{B}$ taking into account of the boundary condition. [(CO4) (Evaluate/HOCQ)]

5 + 7= 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	9.37%	66.66%	23.95%

Course Outcome (CO):

After the completion of the course students will be able to

1. Analyze a given physical problem to construct the requisite mathematical equations with associated initial and boundary conditions if any.
2. Identify the broad category of numerical method to solve the mathematical problem.
3. Justify the appropriateness of the chosen numerical method to solve a particular problem.
4. Execute the algorithm to solve the numerical problem from start to finish to obtain solutions.
5. Evaluate the accuracy and precision of solutions.
6. Ability to troubleshoot solutions in case desired accuracy is not obtained.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
CHE	https://classroom.google.com/c/NDAwNzk0MzgyMzgz/a/NDU0OTI5MzM1OTQw/details