MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If the eigenvalues of a square matrix A are 4 and 6, then the eigen values of A^T are
(a) $\frac{1}{4}, \frac{1}{6}$ (b) $16, 36$ (c) $4, 6$ (d) $\frac{1}{4^2}, \frac{1}{6^2}$.

(ii) Which one of the following sequence is divergent?
(a) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$ (b) $\left\{(1+\frac{1}{n})^n\right\}$
(c) $\left\{\frac{\sin n\pi}{n}\right\}$ (d) $\{2^n\}$

(iii) The series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots \infty$ is
(a) conditionally convergent (b) absolutely convergent
(c) divergent (d) oscillatory.

(iv) Which one of the following is the inverse of the matrix $A = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$?
(a) $\begin{pmatrix} \frac{1}{3} & 0 \\ -\frac{1}{6} & \frac{1}{2} \end{pmatrix}$ (b) $\begin{pmatrix} 0 & \frac{1}{6} \\ -\frac{1}{6} & \frac{1}{2} \end{pmatrix}$ (c) $\begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{2} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

(v) A normal vector to the plane $2x + 3y - z = 0$ is
(a) $2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ (b) $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$
(c) $-2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ (d) $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$.

(vi) Identify the non-linear differential equation amongst the given equations:
(a) $\frac{d^2y}{dx^2} + 4y = 0$ (b) $y\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = 0$
(c) $\frac{dy}{dx} - 3y = 0$ (d) $\frac{d^2y}{dx^2} + (\tan x)\frac{dy}{dx} - 2y = 0$.

(vii) The differential equation $f(x,y)\frac{dy}{dx} + g(x,y) = 0$, is exact if

(a) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ (b) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$ (c) $\frac{\partial f}{\partial x} = -\frac{\partial g}{\partial y}$ (d) $\frac{\partial f}{\partial y} = -\frac{\partial g}{\partial x}$.

(viii) If $f(x,y) = \tan\left(\frac{x}{y}\right)$, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} =?$

(a) $\tan\left(\frac{x}{y}\right)$ (b) $\cot\left(\frac{x}{y}\right)$ (c) 0 (d) $\sec^2\left(\frac{x}{y}\right)$

(ix) The value of $\int_C \frac{1}{x^2}(xdy - ydx)$, where C is the parabola $y = x^2$ from $(1,1)$ to $(2,4)$ is

(a) 1 (b) 0 (c) 3 (d) -1.

(x) $\lim_{y\to 0} \lim_{x\to 0} \frac{x+y}{x-y} =?$

(a) 0 (b) -1 (c) 1 (d) 2.

Group – B

2. (a) Find the eigen values and corresponding eigen vectors of the matrix $\begin{pmatrix} 8 & -4 \\ 2 & 2 \end{pmatrix}$.

[(CO1, CO2) (Evaluate/HOCQ)]

(b) Reduce the matrix A to row-reduced echelon form and hence find its rank where,

$$A = \begin{pmatrix} 1 & -4 & 2 & -1 \\ 3 & -12 & 6 & -3 \\ 2 & -1 & 0 & 1 \\ 0 & 1 & 3 & -1 \end{pmatrix}.$$

[(CO1, CO2) (Understand/LOCQ)]

6 + 6 = 12

3. (a) Determine whether the following system of linear equations is consistent or not.

$$x + y - z = 0$$
$$2x - y + z = 3$$
$$4x + 2y - 2z = 2$$

If consistent, find the solution. [(CO1, CO2)(Apply/IOCQ)]

(b) Verify Caley-Hamilton theorem for the matrix A= $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and hence find A^{-1}.

[(CO1,CO2)(Remember/LOCQ)]

5 + 7 = 12

Group – C

4. (a) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $|\vec{r}| = r$, prove that $\vec{\nabla}f(r) \times \vec{r} = \vec{0}$.

[(CO3, CO4) (Remember/LOCQ)]

(b) Discuss the convergence of the series:

$$1 + \frac{\alpha+1}{\beta+1} + \frac{(\alpha+1)(2\alpha+1)}{(\beta+1)(2\beta+1)} + \frac{(\alpha+1)(2\alpha+1)(3\alpha+1)}{(\beta+1)(2\beta+1)(3\beta+1)} + \cdots \infty \quad (\alpha > 0, \beta > 0).$$

[(CO3, CO4)(Analyze/IOCQ)]

5 + 7 = 12

5. (a) Let a sequence $\{u_n\}$ be defined as, $u_n = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1}$. Show that $\{u_n\}$ is monotonically decreasing and bounded.
[(CO3, CO4)(Understand/LOCQ)]

(b) Evaluate the directional derivative of $(x, y, z) = x^2y^2z^2$at the point $(1, 1, -1)$in the direction of the tangent to the curve $x = e^t, y = \sin 2t + 1, z = t - \cos t$ at $t = 0$.
[(CO3, CO4)(Evaluate/HOCQ)]

6 + 6 = 12

Group – D

6. (a) Solve the ordinary differential equation:$(2x \log x - xy)dy + 2ydx = 0$.
[(CO5) (Understand/LOCQ)]

(b) Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \log x$, $(x > 0)$, by method of variation of parameters.
[(CO5)(Apply/IOCQ)]

6 + 6 = 12

7. (a) Obtain the general solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x$.
[(CO5) (Analyze/IOCQ)]

(b) Solve: $y - 2px - p^n = 0$, where $p = \frac{dy}{dx}$.
[(CO5) (Understand/LOCQ)]

6 + 6 = 12

Group – E

8. (a) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$, then prove that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z}\right)^2 = 2\left(x\frac{\partial u}{\partial x} + \right.$ $y\partial u\partial y + z\partial u\partial z$, where a, b and c are constants.
[(CO6) (Understand/LOCQ)]

(b) If $u = \cos(x^2 + 2y)$ and $= x^4 + 4x^2y + 4y^2$, then find$\frac{\partial(u,v)}{\partial(x,y)}$.
[(CO6) (Remember/LOCQ)]

(c) Show that $\int_C \frac{x^2dy - y^2dx}{x^{5/3}+y^{5/3}} = \frac{3\pi}{16}a^{\frac{4}{3}}$ where C is the quarter of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, in the first quadrant.
[(CO6) (Analyze/IOCQ)]

5 + 1 + 6 = 12

9. (a) Use divergence theorem to evaluate$\iint_S \{xz^2dydz + (x^2y - z^3)dzdx +$ $2xy + y2zdxdy$ where, S is the surface of the hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$.
[(CO6) (Evaluate/HOCQ)]

(b) Apply Green's theorem to find $\oint_C \{\cos x \sin y - xy\}dx + \sin x \cos y dy$, where C is the circle $x^2 + y^2 = 1$.
[(CO6) (Apply/IOCQ)]

6 + 6 = 12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	43.75%	37.5%	18.75%

Course Outcome (CO):

After the completion of the course students will be able to

MATH1101.1: Apply the concept of rank of matrices to find the solution of a system of linear simultaneous
equations.

MATH1101.2: Develop the concept of eigen values and eigen vectors.

MATH1101.3: Combine the concepts of gradient, curl, divergence, directional derivatives, line integrals,
surface integrals and volume integrals.

MATH1101.4: Analyze the nature of sequence and infinite series. .

MATH1101.5: Choose proper method for finding solution of a specific differential equation.

MATH1101.6: Describe the concept of differentiation and integration for functions of several variables with their applications in vector calculus.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
AEIE	https://classroom.google.com/c/NDEwMDU1MTY2NzI2/a/NDY0MTk1Mzg2OTYy/details
AIML	https://classroom.google.com/c/NDA2MTE1NjU4MTAz/a/NDY0MjY1Njk4MTY4/details
BT	https://classroom.google.com/c/NDA0Nzc5NDY1ODQ1/a/NDYzOTc4NjMxMjEw/details
CHE	https://classroom.google.com/c/NDA2MTE2MTMwMTk1/a/NDY0MjU3NTA2MTI0/details
CE	https://classroom.google.com/c/NDA0Nzc5NDY1OTY5/a/NDYzOTgwNTgzMTkw/details
CSBS	https://classroom.google.com/c/NDAxNTAyMjU3NTE0/a/NDY0MjYzMzEyNzg4/details
CSE-A	https://classroom.google.com/c/NDA1Mzc3MTA5MzQ4/a/NDY0MDA5NzQ2OTc0/details
CSE-B	https://classroom.google.com/c/NDA1MzczODEwNTAw/a/NDY0MDEwMzAwNTQ0/details
CSE-C	https://classroom.google.com/c/NDA1Mzc0MDQ5ODQy/a/NDY0MDExODc4NTM5/details
DS	https://classroom.google.com/c/NDA1OTA1OTc4MDQ4/a/NDY0MzQ3OTY0MjI4/details
ECE-A	https://classroom.google.com/c/NDA1MzM3MjAzOTIz/a/NDY0MzU3MjU3NzQy/details
ECE-B	https://classroom.google.com/c/NDA1MzM3MjA0MDA5/a/NDY0MzU3MjU3Njkz/details
ECE-C	https://classroom.google.com/c/NDAxNDkwNjMwMTYw/a/NDY0MjM2NDExMjI5/details
EE	https://classroom.google.com/c/NDAxNDkwNjMwMjAw/a/NDY0MjQzNzIwMTQ1/details
ME	https://classroom.google.com/c/NDA0NDcyMjgxNTc3/a/NDYzOTQ4OTIxOTEy/details
IT	https://classroom.google.com/c/NDA0NDc4NTg2ODg1/a/NDYzOTQ1ODg3OTY5/details

Note: Students having backlog in MATH1101 (new syllabus) are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):

https://classroom.google.com/c/NDY0NTA1OTk5NDY3?cjc=ww7zqm2

Step-II: Submit your answer script by clicking link below:

https://classroom.google.com/c/NDY0NTA1OTk5NDY3/a/NDY0NTA1OTk5NjM1/details