

**DISCRETE MATHEMATICS**  
**(CSEN 2102)**

**Time Allotted : 3 hrs**

**Full Marks : 70**

*Figures out of the right margin indicate full marks.*

*Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.*

*Candidates are required to give answer in their own words as far as practicable.*

**Group - A**  
**(Multiple Choice Type Questions)**

1. Choose the correct alternative for the following: **10 × 1 = 10**
- (i) If  $a$  divides  $c$  and  $b$  divides  $c$  with  $\gcd(a, b) = 1$ , then  
(a)  $c$  divides  $ab$  (b)  $ab$  divides  $c$   
(c)  $\gcd(a, c) = 1$  (d)  $\gcd(a, c) = 2$
- (ii) The integer 5 belongs to the same residue class of integer modulo 7 as  
(a) 13 (b) -13 (c) 9 (d) -9
- (iii) The generating function of the sequence 1, 0, -1, 0, 1, 0, -1, 0, 1, 0, ... is  
(a)  $\frac{1}{(1-x)^2}$  (b)  $\frac{1}{1-x^2}$  (c)  $\frac{1}{1+x^2}$  (d)  $\frac{1}{(1+x)^2}$
- (iv) From a club consisting of 6 men and 7 women, in how many ways can we select a committee of 3 men and 4 women?  
(a) 600 (b) 500 (c) 525 (d) 700
- (v) Four persons enter a railway carriage in which there are six seats; in how many ways can they take their places?  
(a) 24 (b) 240 (c) 360 (d) 120
- (vi) The greatest common divisor of 123456 and 123457 is  
(a) 3 (b) 9 (c) 11 (d) 1
- (vii) A graph is planar if and only if  
(a) it does not contain subgraphs homeomorphic to  $K_5$  and  $K_{3,3}$ .  
(b) it does not contain subgraphs isomorphic to  $K_5$  or  $K_{3,3}$ .  
(c) it does not contain subgraphs isomorphic to  $K_5$  and  $K_{3,3}$ .  
(d) it does not contain subgraphs homeomorphic to  $K_5$  or  $K_{3,3}$ .
- (viii) Which one of the following is not true for the chromatic polynomial  $f(G, x)$  having chromatic no.  $m$ .  
(a)  $f(G, m) \neq 0$  (b)  $f(G, m - 1) \neq 0$   
(c)  $f(G, m + 1) \neq 0$  (d)  $f(G, m + 2) \neq 0$

- (ix) Negation of “All students live in the dormitories” is  
 (a) All students do not live in dormitories  
 (b) No students live in dormitories  
 (c) One student does not live in dormitories  
 (d) Some students do not live in dormitories.
- (x)  $A \wedge B$  is equivalent to which of the following?  
 (a)  $\sim A \rightarrow \sim B$  (b)  $\sim A \rightarrow B$   
 (c)  $\sim B \rightarrow A$  (d)  $\sim (A \rightarrow \sim B)$ .

**Group - B**

2. (a) Find the chromatic polynomial of the following disconnected graph  $G$ . Hence find the chromatic no. of  $G$ .

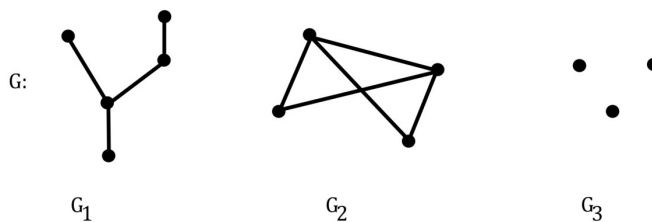


Figure 1

[[CO1] & (CO2) (Evaluate/HOCQ)]

- (b) Let  $G$  be connected planar graph with 9 vertices having degree sequence  $(2, 2, 2, 3, 3, 3, 4, 4, 5)$ . Find the number of edges and regions for  $G$ .

[[CO1] & (CO2) (Apply/IOCQ)]

- (c) Draw a graph having the given properties or explain why no such graph can exist.

- (i) A binary tree having degree sequence  $(1, 2, 2, 3, 3, 3)$ .  
 (ii) A binary tree of height 3 and 10 leaves.

[[CO1] & (CO2) (Understand/LOCQ)]

$$6 + 3 + 3 = 12$$

3. (a) There are four teachers  $t_1, t_2, t_3$  and  $t_4$  who are designated for five workshops  $w_1, w_2, w_3, w_4$  and  $w_5$  as follows:

$$t_1 \rightarrow \{w_1, w_2\}, t_2 \rightarrow \{w_1, w_2, w_5\}, t_3 \rightarrow \{w_1, w_2, w_3, w_5\} \text{ and } t_4 \rightarrow \{w_3, w_4\}.$$

Using Hall’s theorem, find whether it is possible to assign a workshop to every teacher? Solve this problem graphically. [[CO1] & (CO2) (Evaluate/LOCQ)]

- (b) Prove that if  $G$  is self-dual with  $n$  vertices and  $e$  edges then  $e = 2n - 2$ . Draw the dual of the following graph:

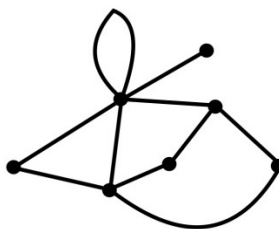


Figure 2

[[CO1] & (CO2) (Apply/IOCQ)]

- (c) Check whether the polynomial  $x^4 + 2x^3 - 4x^2$  is a possible chromatic polynomial of a non-null graph. [(C01) & (C02)(Remember/LOCQ)]

**6 + (2 + 3) + 1 = 12**

**Group - C**

4. (a) Let  $a$  be any integer. Prove that  $a^3 \equiv 0, 1 \text{ or } 6 \pmod{7}$ . [(C03) (Understand/LOCQ)]  
(b) By use of the Euclidean algorithm, solve the following Diophantine equation  $1234x + 981y = 1$ . [(C03) (Evaluate/HOCQ)] **6 + 6 = 12**
5. (a) Find the remainder in the division of  $4^{702} + 5^{403} + 7^{22} + 10!$  by 11. State every theorem that you use and show your calculations in detail. [(C03)(Evaluate/IOCQ)]  
(b) Let  $a$  be any integer. Prove that 42 is a divisor of  $a^7 - a$ . [(C03)(Analyze/IOCQ)] **6 + 6 = 12**

**Group - D**

6. (a) (i) How many committees of 5 or more can be chosen from 9 people?  
(ii) In how many ways can a committee of 5 teachers and 4 students be chosen from 9 teachers and 15 students?  
(iii) In how many ways can the committee in (ii) be formed if teacher  $A$  refuses to serve if Student  $B$  is on the committee? Give explanations of your answers in detail. [(C04) (Analyze/IOCQ)]  
(b) (i) Assuming that repetitions are not permitted, how many four-digit numbers can be formed from the six digits 1, 2, 3, 5, 7, 8?  
(ii) How many of the numbers in part (i) are less than 4000?  
(iii) How many of the numbers in part (i) are even?  
(iv) How many of the numbers in part (i) are odd?  
(v) How many of the numbers in part (i) are multiple of 5?  
(vi) How many of the numbers in part (i) contain both the digits 3 and 5? [(C04)(Understand/LOCQ)] **6 + 6 = 12**
7. (a) Use the method of generating function to solve the recurrence relation  $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$  given that  $a_0 = 2$  &  $a_1 = 8$ . [(C04)(Apply/IOCQ)]  
(b) Find the number of integers from 1 to 250 both inclusive, that are not divisible by any of the integers 2, 3, 5 and 7. [(C04)(Apply/IOCQ)] **7 + 5 = 12**

**Group - E**

8. (a) Prove the following equivalence without using truth tables:

- $(q \wedge \sim (p \wedge q)) \vee (p \wedge (\sim p \vee q)) \equiv q.$  [(C05) & (C06)(Analyze/IOCQ)]
- (b) Prove that the premises  $a \rightarrow (b \rightarrow c), d \rightarrow (b \wedge \sim c)$  and  $a \wedge d$  are inconsistent. [(C05) & (C06) (Evaluate/HOCQ)]
- (c) Find the principal conjunctive normal form of the following statement:  
 $(p \rightarrow (q \wedge r)) \wedge (\sim p \rightarrow (\sim q \wedge \sim r)).$  [(C05) & (C06) (Apply/IOCQ)]  
**4 + 5 + 3 = 12**
9. (a) Prove the following equivalences by proving the equivalences of the duals  
 $(p \wedge (p \leftrightarrow q)) \rightarrow q \equiv T.$  [(C05) & (C06)(Analyze/IOCQ)]
- (b) Find whether the conclusion  $C$  follows from the premises  $H_1, H_2,$  or not where  $H_1: p \rightarrow (q \rightarrow r), H_2: p \wedge q, C: r$  using the truth table.  
 [(C05) & (C06)(Remember/LOCQ)]
- (c) Find DNF of the following statement  $\sim(p \rightarrow (q \wedge r)).$   
 [(C05) & (C06)(Understand/LOCQ)]  
**6 + 4 + 2 = 12**

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17%	53.13%	17.71%

**Course Outcome (CO):**

After the completion of the course students will be able to

- CSEN2102. 1. Interpret the problems that can be formulated in terms of graphs and trees.
- CSEN2102. 2. Explain network phenomena by using the concepts of connectivity, independent sets, cliques, matching, graph coloring etc.
- CSEN2102. 3. Achieve the ability to think and reason abstract mathematical definitions and ideas relating to integers through concepts of well-ordering principle, division algorithm, greatest common divisors and congruence.
- CSEN2102. 4. Apply counting techniques and the crucial concept of recurrence to comprehend the combinatorial aspects of algorithms.
- CSEN2102. 5. Analyze the logical fundamentals of basic computational concepts.
- CSEN2102. 6. Compare the notions of converse, contrapositive, inverse etc in order to consolidate the comprehension of the logical subtleties involved in computational mathematics.

Department & Section	Submission Link
CSBS	<a href="https://classroom.google.com/c/NDA0Nzk00TcwNzIx/a/NDY0MzUzZmE4MDQy/details">https://classroom.google.com/c/NDA0Nzk00TcwNzIx/a/NDY0MzUzZmE4MDQy/details</a>
CSE - A	<a href="https://classroom.google.com/c/NDA00Tk1NzIxNzY4/a/NDYzOTQ3NzUwMjYy/details">https://classroom.google.com/c/NDA00Tk1NzIxNzY4/a/NDYzOTQ3NzUwMjYy/details</a>
CSE - B	<a href="https://classroom.google.com/c/NDA00Tk1NzIxOTE5/a/NDYzOTQ3NzUwNjAx/details">https://classroom.google.com/c/NDA00Tk1NzIxOTE5/a/NDYzOTQ3NzUwNjAx/details</a>
CSE - C	<a href="https://classroom.google.com/c/NDA00Tk1NzIxOTM5/a/NDYzOTQ3NzUwNTgy/details">https://classroom.google.com/c/NDA00Tk1NzIxOTM5/a/NDYzOTQ3NzUwNTgy/details</a>
IT	<a href="https://classroom.google.com/c/NDA1MTAwNDM4NjQy/a/NDY0MzQ5MjMwNzU1/details">https://classroom.google.com/c/NDA1MTAwNDM4NjQy/a/NDY0MzQ5MjMwNzU1/details</a>

**Note: Students having backlog in CSEN2102 are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:**

**B.TECH/CSBS/CSE/IT/3<sup>RD</sup> SEM/CSEN 2102/2021**

**Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):**

<https://classroom.google.com/c/NDY0NTA4OTcyODg1?cjc=7yt4sf7>

**Step-II: Submit your answer script by clicking link below:**

<https://classroom.google.com/c/NDY0NTA4OTcyODg1/a/NDU1MTYzNDc1NTE1/details>