NUMERICAL AND STATISTICAL METHODS
(MATH 2002)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Newton-Raphson method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is

(a) $x_{n+1} = \frac{x_n}{2}$ (b) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$

(c) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{R}{x_n}\right)$ (d) $x_{n+1} = \frac{3x_n}{2}$.

(ii) Simpson's 1/3 rule requires

(a) even number of ordinates (b) odd number of ordinates

(c) even or odd number of ordinates (d) prime number of ordinates.

(iii) A system of equation $AX = B$, where $A = [a_{ij}]_{n\times n}$ is said to be diagonally dominant if

(a) $|a_{ii}| > \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|$ for all i (b) $|a_{ii}| < \sum_{\substack{j=1 \\ j\neq i}}^{n} |a_{ij}|$ for all i

(c) $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|$ for all i (d) $|a_{ii}| < \sum_{j=1}^{n} |a_{ij}|$ for all i.

(iv) If a root of an equation $3x^3+5x-40=0$ lies in $(2,3)$, then the interval of length 0.25 within which this root lie is

(a) $(2, 2.25)$ (b) $(2.25, 2.50)$

(c) $(2.50, 2.75)$ (d) $(2.75, 3)$.

(v) In Lagrange's polynomial the sum of the coefficients of $y_0, y_1, \ldots\ldots, y_n$ i.e., the sum of the Lagrangian coefficients is

(a) 0 (b) 1 (c) 2 (d) 3.

(vi) A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$ (c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

(vii) If a *p.d.f.* is of the form $f(x) = ke^{-\alpha|x|}, x \in (-\infty, \infty)$, then the value of k is

(a) 0.5 (b) 1 (c) 0.5α (d) α.

(viii) The standard deviation of a uniformly distributed random variable between 0 and 1 is

(a) $1/\sqrt{12}$ (b) $5/\sqrt{12}$ (c) $1/\sqrt{3}$ (d) $7/\sqrt{12}$.

(ix) The two lines of regression are given as $x + 2y - 5 = 0$ and $2x + 3y = 8$.
Then the mean values of X and Y are respectively

(a) 2, 1 (b) 1, 2 (c) 2, 5 (d) 2,3.

(x) The mean of Binomial distribution $B\left(10, \frac{2}{5}\right)$ is

(a) 4 (b) 6 (c) 5 (d) 0.

Group–B

2. (a) Explain the geometrical interpretation of Newton-Raphson method. Use this method to evaluate the smallest positive root of the equation $x = \sin x - 0.25$ correct to three decimal places.

(b) Solve the following system of equations by LU-factorization method:

$$x - 5y + z = 2$$
$$2x + 4y + z = 1$$
$$x + y + z = 0$$

(2 + 4) + 6 = 12

3. (a) Solve the following system of equation by using Gauss Elimination method

$$6x - y - z = 19$$
$$3x + 4y + z = 26$$
$$x + 2y + 6z = 22$$

(b) Find the real root of the equation $2x - \cos x - 3 = 0$ by bisection method correct up to two places of decimal.

6 + 6 = 12

Group - C

4. (a) From the following table of $\sin x$ compute $\sin 2^0$.

x^0	0	10	20	30	40
$\sin x^0$	0	0.17365	0.34202	0.50000	0.64279

(b) Compute $y(0.4)$, by Runge-Kutta method of 4th order correct to five decimal places, from the equation $\frac{dy}{dx} - 1 = xy$, with $y(0) = 1, h = 0.1$.

6 + 6 = 12

5. (a) Find the value of $\int_0^{10} \frac{1}{(1+x^2)^{3/2}} dx$ by Simpson's 1/3 rule, taking $n = 6$.

(b) Find the interpolating polynomial by suitable interpolation for the following data.

x:	0	1	4	5
$f(x)$:	-1	7	26	124

Hence find $f(0.5)$.

6 + 6 = 12

Group - D

6. (a) There were three candidates A, B, C for the position of a manager whose chances of getting the appointment are in the proportion $4{:}2{:}3$ respectively. The Probability that A, if selected, would launch a new product in the market is 0.3. The probabilities of B and C doing the same are 0.5 and 0.8 respectively. What is the probability that the new product was launched in the market by C?

(b) The probability mass function of a discrete random variable *X* is defined as

x	0	1	2
$P(X = x)$	$3k^2$	$4k - 10k^2$	$5k - 1$

(i) Find the value of k.

(ii) Obtain the distribution function, $F(x)$ of *X*.

7 + (2 + 3) = 12

7. (a) The demand of a new product of a company is assumed to be a random variable with probability density function

$$f(x) = \begin{cases} xe^{-x^2/2}, & x \geq 0 \\ 0, & elesewhere \end{cases}$$

Find (i) expected demand quantity and (ii) probability that the demand exceeds 1.

(b) A and B throw alternatively with a pair of balanced dice. A wins if he throws a sum of six points before B throws a sum of seven points, while B wins if he throws seven points before A throws a sum of six points. If A begins the game, find the probability of his wining.

(3 + 3) + 6 = 12

Group – E

8. (a) A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with the average number of demand per day 1.5. Calculate the proportion of days on which some demand is refused.

(b) The distribution of height of men is normally distributed with mean 64.5 inches and standard deviation 4.5 inches. Among 10,000 men find the expected number of people with height

(i) less than 69 inches but greater than 55.5 inches. and (ii) less than 55.5 inches.

[Given $\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 0.3413$ and $\int_0^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 0.4772$.]

5 + 7 = 12

9. (a) For a group of 200 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on it was found that the scores 43 and 35 were misread as 34 and 53 respectively. Find the corrected mean and standard deviation corresponding to the corrected figures.

(b) Marks obtained by 10 students in the college test (X) and the university test (Y) are as follows:

X	41	45	50	68	47	77	90	100	80	100
Y	60	63	60	48	85	56	53	91	74	98

Apply the method of regression to estimate the marks a student could have obtained in the university test if he had scored 60 in the college test but was ill at the time of the university test.

5 + 7 = 12

Students having backlog in MATH2002 (old syllabus) are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):
https://classroom.google.com/c/NDc0ODUxODA1MDgw?cjc=s7jljyf

Step-II: Submit your answer script by clicking link below:
https://classroom.google.com/c/NDc0ODUxODA1MDgw/a/NDc0ODUxODA1MTM4/details