MATHEMATICS - I
(MATH 1101)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The following system of equations
$2x_1 - x_2 + 3x_3 = 1$
$3x_1 - 2x_2 + 5x_3 = 2$
$-x_1 + 4x_2 + x_3 = 3$
has
(a) no solution.
(b) a unique solution.
(c) more than one but a finite number of solutions.
(d) an infinite number of solutions.

(ii) If $A = \begin{bmatrix} 8 & 5 \\ 7 & 6 \end{bmatrix}$, then the value of $|A^{121} - A^{120}|$ is
(a) 0. (b) 1. (c) 121. (d) 120.

(iii) A matrix A has dimension 2×2. If the eigen values of the matrix are 1 and 4, what would be the eigen values of A^2.
(a) 1 and 2. (b) 1 and 16. (c) 0.5 and 2. (d) 2 and 4.

(iv) A vector field which has a vanishing divergence is called as
(a) Solenoidal field. (b) Rotational field.
(c) Hemispheroidal field. (d) Irrotational field.

(v) The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$ is convergent for
(a) all real value o f x. (b) $|x| < 1$.
(c) $|x| \leq 1$. (d) $-1 < x \leq 1$.

(vi) If $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$, then the value of the series $\sum_{n=1}^{\infty} \left(\frac{2}{n}\right)^4$ is
(a) $\frac{\pi^4}{45}$. (b) $\frac{8\pi^4}{45}$. (c) $\frac{8\pi^4}{45} - 1$. (d) $16\left(\frac{\pi^4}{90} - 1\right)$.

(vii) Which of the following is not a necessary condition for Cauchy's Mean Value Theorem?
(a) The functions, f(x) and g(x) be continuous in [a, b]
(b) There exists a value $c \in (a,b)$ such that $g'(c) = 0$
(c) The functions f(x) and g(x) be derivable in (a, b)
(d) There exists a value $c \in (a,b)$ such that, $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$.

(viii) Which of the following is a Maclaurin's series?
(a) $\sum_{n=0}^{\infty}(x-2)^n$ (b) $\sum_{n=0}^{\infty}(x-1)^n$
(c) $\sum_{n=0}^{\infty} x^n$ (d) $\sum_{n=0}^{\infty}(x+1)^n$

(ix) If z is homogenous function of degree 2 then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to
(a) z. (b) z^2. (c) 2z. (d) 2.

(x) The volume of a region is obtained by evaluating
(a) line integral. (b) double integral.
(c) triple integral. (d) None of these.

Group – B

2. (a) Find the rank of the following matrix using echelon form. $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & 1 \end{bmatrix}$

(b) Show that $x + 2y - z = 3, 3x - y + 2z = 1, 2x - 2y + 3z = 2$, and $x - y + z = -1$ areconsistent and solve them.

6 + 6 = 12

3. (a) Express the given matrix A as a sum of a symmetric and skew symmetric matrices. where $A = \begin{bmatrix} 2 & -4 & 9 \\ 14 & 7 & 13 \\ 9 & 5 & 11 \end{bmatrix}$.

(b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and hence find A^{-1}.

6 + 6 = 12

Group – C

4. (a) Show that there is no real number A for which the equation $x^3 - 27x + A = 0$ has two distinct roots in [0,2].

(b) Assuming the validity of expansion, find the series expansion of $f(x) = e^{2x}$ for all real values of x.

6 + 6 = 12

5. (a) Check the convergence of the following series

(i) $\sum \frac{n^3}{1+2^n}$ (ii) $\sum \left(\frac{n+1}{2n-1}\right)^n$

(b) Test the absolute and conditional convergence of the series $\sum \left(\frac{(-1)^{n+1}}{2n+1}\right)$.

6 + 6 = 12

Group – D

6. (a) Find the nth derivative of $x^2 \log x$.

(b) Locate the critical points of the function $f(x, y) = x^2y^2 - x^2 - y^2$ and classify them as relative minimum, relative maximum and saddle points.

6 + 6 = 12

7. (a) Discus the limit of the following functions at the point (0,0).

(i) $f(x, y) = \frac{xy}{x^2+y^2}$ for $(x, y) \neq (0,0)$.

(ii) $f(x, y) = \frac{(y-x)(1+x)}{(y+x)(1+y)}$, for $x + y \neq 0, -1 < x, y < 1$.

(b) If $u = \tan^{-1} \frac{x^3+y^3}{x-y}$ then prove that $x\frac{du}{dx} + y\frac{du}{dy} = \sin 2u$.

6 + 6 = 12

Group – E

8. (a) Evaluate $\int_0^1 \int_o^{\sqrt{1+x^2}} \frac{1}{1+x^2+y^2} dy\, dx$.

(b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2$ at $(2, -1, 2)$ in the direction of $2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$.

6 + 6 = 12

9. (a) If $\vec{a} = xy\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$, then show that $Curl(Curl\ \vec{a}) = 3\ \hat{\jmath}$.

(b) Using Green's formula, evaluate the line integral

$\oint_C (x - y)dx + (x + y)dy$,

Where C is the circle $x^2 + y^2 = a^2$.

6 + 6 = 12

Students having backlog in MATH1101 (old syllabus) are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):
https://classroom.google.com/c/NDY0NTA1OTk5NDY3?cjc=ww7zqm2

Step-II: Submit your answer script by clicking link below:
https://classroom.google.com/c/NDY0NTA1OTk5NDY3/a/MjI3ODkwMTQ5MjEw/details