DIGITAL SIGNAL PROCESSING
(ELEC 3141)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The digital frequency(Ω)and fundamental period (N) of the signal $x(n) = 5 + \cos(0.5n + 30^0)$, if any, are
(a) $\Omega = 0.5$, $\mathrm{N} = 0$ (b) $\Omega = \pi,\ N = 2$
(c) $\Omega = 0.5\pi,\ \mathrm{N} = 5$ (d) $\Omega = 5,\ N = 0$

(ii) A fraction delay signal $g(n) = x(n - \alpha)$ can be generated from an original signal $x(n)$ using onethe following sequence of operations
(a) up-sampling, low-pass filter, and down-sampling
(b) up-sampling, delay, and down-sampling
(c) up-sampling, and down-sampling
(d) down-sampling, delay, up-sampling

(iii) Impulse response of a LTI discrete time system (FIR-filter) described by $y(n) = 2x(n) + 5x(n-1) - 10x(n-4)$, where $x(n) = \delta(n)$is
(a) $h(n) = (3, -10,\ 5,\ 7, -6);\ \ n = 0\ \ to\ 4$
(b) $h(n) = (0,\ 2,\ 5,\ -10,\ 3);\ \ n = 0\ to\ \ 4$
(c) $h(n) = (2,\ 5,\ 0,\ 0, -10);\ \ \ n = 0\ to\ \ 4$
(d) $h(n) = (2,\ 5, -10,\ 5,\ 7\ );\ \ n = 0\ \ to\ 2$

(iv) The z-transform of a signal $X(z) = \frac{z}{z-a} + \frac{z}{z-b}$ with ROC $\{|z| > |a|\} \cap \{|z| < |b|\}$ converges if
(a) $|b| < |a|$ (b) $|b| = |a|$ (c) $\frac{|a|}{|b|} > 2$ (d) $|b| > |a|$

(v) The impulse response of the difference equation $\mathrm{y(n)} - 2\mathrm{x(n-1)} + 5\ \mathrm{x(n-2)} = 3\ \mathrm{x(n)}$ for right sided input signal $\mathrm{x(n)} = \delta\mathrm{(n)}$is
(a) $\mathrm{y(n)} = \{1, -7\ ,\ 2\}$;for n = 0 to 2 (b) $\mathrm{y(n)} = \{1,\ -5,\ \ 2\}$; for n = 0 to 2
(c) $\mathrm{y(n)} = \{1, -2\ ,\ 5\ \}$; for n = 0 to 2 (d) $\mathrm{y(n)} = \{3, 2, -5\ \}$; for n = 0 to 2

(vi) The DFT coefficient $X(1)$ of the four-point segment $x(0) = 1,\ x(1) = 0,\ x(2) = 0\,,\ x(3) = 1$ of a sequence $x(n)$ is

(a) $X(1) = 0$ (b) $X(1) = 1 - j$

(c) $X(1) = 1 + j$ (d) $X(1) = 1 + j2$

(vii) If the DFT of 4-point sequence of signal $x(n)$is $X(k) = \{4, -j2\,,\ 0,\ \ j2\};\ for\ k = 0\ to\ 3$, then DFT of $g(n) = x(n-2)$ is

(a) $G(k) = \{1,\ j4\,,\ 2,\ \ -j4\};\ for\ n = 0\ to\ 3$

(b) $G(k) = \{4,\ j2\,,\ 0,\ \ -j2\};\ for\ n = 0\ to\ 3$

(c) $G(k) = \{0,\ j2\,,\ 4,\ \ -j2\};\ for\ \ n = 0$ to 3

(d)$G(k) = \{1,\ -j\,,\ 4,\ -j\};\ \ for\ n = 0\ to\ 3$

(viii) Let $x(n) = \{1,\ 2,\ \ 0,\ \ 3\}$ for $n = 0\ to\ 3$. The circularly folded signal $x(-n)$ is

(a) $\{1,\ 3,\ 0,\ 2\}$; for $n = 0\ to\ 3$ (b) $\{1,\ 3,\ 0,\ 2\}$; for $n = 0\ to\ 3$

(c)$\{3,\ 0,\ 1,\ 2\}$; for $n = 0\ to\ 3$ (d) $\{\ 0,\ 3,\ 1, 2\}$; for $n = 0\ to\ 3$.

(ix) If, a causal and stable discrete time system $H(z) = \frac{z}{z+0..2}$ is excited with a sinusoidal input $x(n) = \cos(0.05\pi n)\,u(n)$ then the gainof the system $(H(z))$ is

(a) 0.7 (b) 0.84 (c) 0.95 (d) 0.45

(x) The transfer function $H(z) = \frac{(0.15z^2+0.8z+1)}{(z^2+0.8z+0.15)}$ is a description of

(a) Low-pass filter (b) band-pass filter

(c) all pass filter (d) band- stop filter

Group - B

2\. (a) Explain the basic function of each component of a digital signal processing unit with a block diagram. Show the nature of the signal at the output of each block while the input to the $'DSP'$ unit is $x(t) = e^{-2t}u(t)$. [(CO1) (Low-Order Cognitive/LOCQ)]

(b) Consider a signal $\mathrm{x(n) = u(n) - u(n-5)}$ where $\mathrm{u(n)}$ is a unit step sequences. Find and sketch its odd and even parts. Is this signal $\mathrm{x(n)}$ energy signal or power signal? Find its numerical value.[(CO1) (Intermediate -Order Cognitive /IOCQ)]

(c) Show the sequence of operations required to generate $\mathrm{z(n) = x\left(n - \frac{2}{3}\right)}$ from $\mathrm{x(n) = \{1 \quad 4 \quad 7 \quad 10 \quad 13\}}$; forn = 0, to 4. Step interpolation is considered where required. Sketch the new signal z(n).[(CO1) (High-Order Cognitive /HOCQ)]

4 + 5 + 3= 12

3\. (a) A non-recursive filter of length $(M+1)$ is described by $y(n) = b_o x(n) + b_1 x(n-1) + \ b_2 x(n-2) + \cdots \ldots + b_M x(n-M)$. Show that the impulse response sequence or samples are the filter 'tap-coefficients'(i.e$b_i = 0,\ 1,\ 2, \ldots \ldots . M$)[(CO1) (Low-Order Cognitive/LOCQ)]

(b) Two discrete time systems are connected in parallel and impulse response of each system is described as:System-1: $\mathrm{h}_1(n) = \{3,\ 2,\ 4\};\ for\ 0 \le n \le 2$ and System-2: $\mathrm{h}_2(n) = \{-2,\ 3,\ 0\};\ for\ \ 0 \le n \le 2$. Find the output sequence y(n) of the system using the linear convolution sum with an input signal $x(n) = \{1 \quad 4 \quad 3\}$;for $0 \leq n \leq 2$.[(CO1) (Intermediate-Order Cognitive/IOCQ)]

(c) Find the zero-input response of the discrete time system $2y(n) - \frac{1}{3}y(n-1) - \frac{1}{3}y(n-2) = x(n),\ n \geq 0,$ with $y(-1) = 0$ and $y(-2) = 12$. Obtain the values of $y(n)$ for $n = 2,\ 4$. [(CO1) (High-Order Cognitive/HOCQ)]

3 + 6 + 3= 12

Group - C

4. (a) Determine the $z-$transform of the signal $x(n) = (0.8)^n\ u(n);$ for $n = 0,\ 1,\ 2,\ 3$ and its R.O.C. [(CO2) (Low-Order Cognitive /LOCQ)]

(b) Realize the following digital filter (system) using "direct-form-II" structure$y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$. Is the digital filter stable?

[(CO2) (Intermediate Order Cognitive /IOCQ)]

(c) For the transfer function $H(z) = \frac{z^2}{(z^2 - 4z + 5)}$ use the initial value and final value theorems to determine $x(0)$ and $x(\infty)$ (Justify your answer).

[(CO2) (High-Order Cognitive/HOCQ)]

3 + 6 + 3= 12

5. (a) Let $X(z) = z-$transform of $x(n) = Z\{x(n)\}$ with ROC: R_x. Derive the expression to find the $z-$transform of delayed signal $y(n) = x(n-m)$ from the basic definition of $z-$transform and its ROC. [(CO2)(Low-Order Cognitive/LOCQ)]

(b) Determine the inverse $z-$transform of $X(z) = \frac{3z^2 - 1}{(z^2 - 3z + 2)}$; ROC: $1 < |z| < 2$ using partial fraction method. Is the given system stable for a unit impulse input?

[(CO2)(Intermediate-Order Cognitive/IOCQ)]

(c) Consider a linear system with the linear difference equation
$y(n) = y(n-1) - 0.5y(n-2) + 3x(n) - 2x(n-1)$.
Find the$z-$transfer function of the discrete time system and also sketch poles and zeros in the $z-$plane.[(CO2) (High-Order Cognitive/HOCQ)]

4 + 5 + 3 = 12

Group – D

6. (a) Define DFT of a finite sequence$\{x(n)\}$. Show that the $N-point\ DFT$ of a finite sequence signal $x(n)$ is periodic with period $'N'$.

[(CO3) (Low-Order Cognitive/LOCQ)]

(b) Given a sequence $x(n) = \{1,\ 2,\ 3\ ,4\}$for $0 \leq n \leq 3$, evaluate its DFTcoefficients$X(k)$. [(CO3) (Intermediate-Order Cognitive/IOCQ)]

(c) Consider a 4-point DFT coefficients of a signal $(x(n), n = 0\ ,1, 2, 3)$ are known.Compute the amplitude spectrum, Phase spectrum and power spectrum for the

DFTcoefficient X(2)= -2 only. Discuss the significance of the DFTcoefficients$X(2)$ in context of the signal x(n). Assuming that sampling frequency $f_s = 200\ Hz$.[(CO3) (High-Order Cognitive/HOCQ)]

4 + 5 + 3= 12

7. (a) Which concept is used to account for the inherent periodicity of DFT and IDFT and why? [(CO3) (Low-Order Cognitive /LOCQ)]

(b) Find the signal $x(n)$ from the DFT coefficients $X(k) = \{10, \ -2 + j2, \ -2, \ -2 - j2,$ for $n=0, 1, 2, 3$.[(CO3) (Intermediate-Order Cognitive/IOCQ)]

(c) Let $x(n) = \{1, -2.\ 5,\ 3,\ 2,\ \text{-}5\}$ for n = 0 to 5is a periodic extension of$N = 6$ point signal. Find the value of the expression $\mathrm{V} = \mathrm{W}_6^0 + \mathrm{W}_6^1 + \mathrm{W}_6^2 + \mathrm{W}_6^3 + \mathrm{W}_6^4 + \mathrm{W}_6^5$, where $\mathrm{W}_\mathrm{N} = \mathrm{e}^{-\mathrm{j}\frac{2\pi}{\mathrm{N}}}$ is the complex quantity, known as the twiddle factor. Show the twiddle factor W_N periodic and find its periodicity.

[(CO3) (High-Order Cognitive/HOCQ)]

3 + 6 + 3= 12

Group - E

8. (a) What do you mean by linear phase FIR −filter? Discuss a simple method to check whether a FIR system or filter is linear phase. [(CO4) (Low-order Cognitive/LOCQ)]

(b) The impulse response h(n) of a filter is given$\mathrm{h}(0) = 1, \mathrm{h}(1) = 2,\ \mathrm{h}(2) = 2,\ \mathrm{h}(3) = 1$.Is it a linear-phase filter FIR? If so, of what type. What is the delay(α) in the response of the filter?[(CO4) (Intermediate-Order Cognitive/IOCQ]

(c) Consider anFIR linear-phase filter with impulse response $\mathrm{h(n) = u(n) - u(n-4)}$. Sketch the pole-zero plot of H(z).[(CO4) (Higher-Order Cognitive/HOCQ)]

4+5+3 =12

9. (a) Describe advantages of using FIR filters over the IIR filters.

[(CO4)(Lower-Order Cognitive/LOCQ)]

(b) Transform the given analog filter $H_c(s) = \frac{5(s+2)}{(s+3)(s+4)}$ into a digital filter $H(z)$ using $\boldsymbol{Bilinear\ Z - transformation}$ with $T = 2sec.$. Is the filter stable and linear phase? [(CO4) (Intermediate-Order Cognitive/IOCQ)]

(c) The transfer function of digital filter H(z) is given by

$$H(z) = \frac{(0.0403\ +\ 0.1208\, z^{-1} +\ 0.1208\, z^{-2} + 0.0403 z^{-3})}{(1 -\ 1.4726 z^{-1} + 1.1715\, z^{-2} -\ 0.3767 z^{-3})}$$

Check whether the given filter will act as a low-pass or high-pass filter (without calculating the magnitude response of the filter) by calculating the filter gains only at low frequency and high frequency. [(CO4) (Higher-Order Cognitive/HOCQ]

3+ 6 + 3 =12

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	29.17%	45.83%	25%

Course Outcome (CO):

After the completion of the course students will be able to

CO1: Understand, interpret, represent, manipulate, process, and analyze of discrete time signalsand systems in the context of digital signal processing.

CO2: Understand a new representation of signal sequences with the z-transform, concept oftransfer-function, and an application of z-transform properties for modelling of discretetime signals and stability analysis of systems.

CO3: Understand the frequency domain analysis of discrete time signals, spectral analysis andexistence of efficient and fast algorithms for DSP systems.

CO4: Understand the design and analyze for digital filters, concept of linear-phase filters,realization of filter structures, mapping from analog filter to digital filter, andimplementation of digital filters in real time (with Digital signal processor).

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	Submission Link
EE	https://classroom.google.com/c/NDA1MDkxNTUxNTE4/a/NDYzMDEzMzA1Mjc0/details