MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of the integral $\oint_C \frac{z^2-z+1}{z-2}dz$ where $C:|z| = 1$ is
(a) 0 (b) $2\pi i$ (c) $-2\pi i$ (d) 1.

(ii) The function $f(z) = \frac{1}{z}$ is analytic
(a) at $z = 0$ (b) at $z = 1$
(c) at all points (d) Nowhere.

(iii) The period of $\cos 2\pi x$ is
(a) 2π (b) 1 (c) 2 (d) 3.

(iv) If $f(x) = x\sin x, -\pi \leq x \leq \pi$, be presented in Fourier series as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos nx + b_n \sin nx]$$

Then the value of a_0 will be
(a) 2 (b) 4 (c) 0 (d) 1.

(v) The value of $P_0(x)$ is
(a) -1 (b) 1 (c) 0 (d) x.

(vi) The value of $J_0(0)$ is
(a) $\frac{1}{2}$ (b) -1 (c) 1 (d) 0.

(vii) $J_{\frac{1}{2}}(x) =$
(a) $\sqrt{\frac{2\pi}{x}}\cos x$ (b) $\sqrt{\frac{2\pi}{x}}\sin x$
(c) $\sqrt{\frac{2}{\pi x}}\cos x$ (d) $\sqrt{\frac{2}{\pi x}}\sin x$.

(viii) The function $f(z) = \bar{z}$ is
(a) continuous at $z = 0$ (b) differentiable at $z = 0$
(c) analytic (d) not continuous at $z = 0$.

(ix) The order and the degree of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2$ is
(a) 2, 2 (b) 1, 2 (c) 2, 1 (d) 1,1.

(x) The value of the integral $\oint_C \frac{dz}{z-1}$ where C is the circle $|z| = 2$ is
(a) 0 (b) -1 (c) πi (d) $2\pi i$.

Group–B

2. (a) State Cauchy's Integral formula. Use it to evaluate the integral $\oint_C \frac{z}{(z^2-1)} dz$ where C is the circle $|z| = 2$.
(b) Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is a harmonic function and determine the corresponding analytic function $u + iv$.

6 + 6 = 12

3. (a) Show that the function $f(z) = \begin{cases} \frac{2xy^2}{x^3+4y^3}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ is not continuous at $z = 0$.
(b) Determine the poles and residues at each of the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$.

6 + 6 = 12

Group - C

4. (a) Find the Fourier series expansion of the function $f(x) = \frac{\pi - x}{2}$ in $0 \leq x \leq 2\pi$.
(b) Find the Fourier sine transform of the function $f(x) = \begin{cases} x, & 0 < x < a \\ 0, & x > a \end{cases}$.

6 + 6 = 12

5. (a) Find the half-range Fourier (i) sine series and (ii) cosine series of the function $f(x) = x$ in the interval $0 < x < 2$.
(b) Find the Fourier transform of the function $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$.

6 + 6 = 12

Group - D

6. (a) Solve in series the equation $y'' + xy = 0$ about the point $x = 0$.
(b) Show that $P_n(-1) = (-1)^n$.

8 + 4 = 12

7. (a) Show that $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$. Hence show that $xJ'_n(x) = -nJ_n(x) + xJ_{n-1}(x)$.

(b) Show that $\int_{-1}^{1} P_m(x)P_n(x)$=0 when $\neq n$.

6 + 6 = 12

Group – E

8. (a) Form the partial differential equation from the relation $2z = (ax + y)^2 + b$, where a and b are arbitrary constants.

(b) Use Charpit's method to solve the partial differential equation $px + qy = pq$.

5 + 7 = 12

9. (a) Solve the partial differential equation:$(mz - ny)p + (nx - lz)q = ly - mx$.

(b) Use method of separation of variable to solve the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, subject to the condition $u(x, 0) = 6e^{-3x}$.

6 + 6 = 12

Students having backlog in MATH2001 (old syllabus) are advised to follow the steps as mentioned below in order to submit the answer-scripts properly:

Step-I: Join the Google classroom by clicking the following link (note that you have to join using your institutional email account):

https://classroom.google.com/c/NDY0NTA4MTU3MjY4?cjc=yslwkq2

Step-II: Submit your answer script by clicking link below:

https://classroom.google.com/c/NDY0NTA4MTU3MjY4/a/NDc0ODUxNTczMDcz/details