METHODS IN OPTIMIZATION
(MATH 4121)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of λ for which the following pay-off matrix is strictly determinable is

	PLAYER B		
PLAYER A	λ	7	3
	−2	λ	−8
	−3	4	λ

(a) $\lambda \le -2$ (b) $-2 \le \lambda \le 3$
(c) for any value of λ (d) $\lambda \ge 7$.

(ii) The quadratic form $Q(x, y, z) = x^2 + y^2 + z^2 + 3yz$ is
(a) positive definite (b) positive semi definite
(c) negative definite (d) indefinite.

(iii) For the standard minimization problem, the Kuhn-Tucker necessary conditions are also sufficient conditions, provided
(a) objective function is convex and constraints are concave
(b) objective function and constraints are concave
(c) objective function and constraints are convex
(d) objective function is concave and constraints are convex.

(iv) The optimal solution of the following game problem is

	PLAYER B			
PLAYER A		B_1	B_2	B_3
	A_1	15	2	3
	A_2	6	5	7
	A_3	−7	4	0

(a) (A_3,B_2) (b) (A_2,B_2) (c) (A_1,B_1) (d) (A_3,B_3).

(v) The bordered Hessian matrix of the Lagrange function L constructed for an optimization problem with equality constraints is given by

$$H^B(L) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 1 & 2 & 2 & 0 \\ 1 & 3 & 0 & 2 \end{pmatrix}$$

then the number of equality constraints involved in the problem is,

(a) 1 (b) 2 (c) 3 (d) 0.

(vi) A game is said to be fair game if the maximin and minimax values of the game are:

(a) equal (b) both equal to one
(c) both equal to zero (d) not equal.

(vii) The solution $x_1 = 1, x_2 = 2$ and $x_3 = 3$ of the system

$$2x_1 + 2x_2 + 8x_3 = 30$$
$$x_1 + 4x_3 = 13$$
$$x_1 + 2x_2 + 4x_3 = 17$$

is

(a) a basic solution (b) a non-basic solution
(c) a degenerate basic feasible solution (d) a non-feasible basic solution.

(viii) If a constant be added to any row or any column of the cost matrix of an assignment problem, then

(a) the resulting assignment problem has the same optimal solution as the original problem
(b) the resulting assignment problem has a different optimal solution from the original problem
(c) the resulting assignment problem will not give an optimal solution
(d) the optimal solution of the resulting assignment problem can be obtained by adding same constant to the optimal solution of the original problem.

(ix) In an $m \times n$ transportation problem, a basic feasible solution will contain

(a) at most $m + n - 1$ positive variables
(b) at most mn positive variables
(c) at most $mn - 1$ positive variables
(d) at most $mn - (m + n - 1)$ positive variables.

(x) In Golden section search algorithm, if the initial interval of uncertainty is L_0, then the length of the final interval L_j is,

(a) $L_j = \frac{1}{\gamma^{j+2}} L_0$ (b) $L_j = \frac{1}{\gamma^{j}} L_0$
(c) $L_j = \frac{1}{\gamma^{j+1}} L_0$ (d) $L_j = \frac{1}{\gamma^{j-1}} L_0$

(where γ is the golden ratio)

Group – B

2. (a) A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods A and B are available at a cost of Rs.4

and Rs. 3 per unit respectively. One of A contains 200 units of vitamins, 1 unit of mineral and 40 calories and one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 calories. Formulate this problem as an LPP model and solve it by graphical method to find combination of foods to be used to have least cost? [(CO1, CO2) (Create/HOCQ)]

(b) Solve the following LPP by Simplex method:
Maximize $z = 3x_1 + 2x_2$
subject to the constraints

$$x_1 - x_2 \leq 5$$
$$2x_1 - 3x_2 \leq 30$$
$$x_1, x_2 \geq 0.$$

[(CO1, CO2) (Apply/IOCQ)]

7 + 5 =12

3. (a) Solve the following LPP by Big-M method:
Maximize $z = 3x_1 + 2x_2 + 3x_3$
subject to the constraints

$$2x_1 + x_2 + x_3 \leq 2$$
$$3x_1 + 4x_2 + 2x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0.$$

[(CO1, CO2) (Apply/IOCQ)]

(b) Obtain the dual LP problem of the following primal LP problem
Minimize $z = x_1 + 2x_2$
subject to the constraints

$$2x_1 + 4x_2 \leq 160$$
$$x_1 - x_2 = 30$$
$$x_1 \geq 10$$
$$x_1,\ x_2 \geq 0$$

[(CO1, CO2) (Understand/LOCQ)]

7 + 5 = 12

Group – C

4. (a) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
Demand	20	40	30	10	

[(CO1, CO2, CO3, CO4)(Evaluate/HOCQ)]

(b) Solve the following assignment problem:

	I	*II*	*III*	*IV*
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	8	15	11	9

[(CO1, CO2, CO3, CO4)(Evaluate/HOCQ)]

7 + 5 = 12

5. (a) Use algebraic method to solve the following game:

	PLAYER B		
PLAYER A	9	1	4
	0	6	3
	5	2	8

[(CO1, CO2, CO3, CO4) (Apply/IOCQ)]

(b) Use graphical method in solving the following game and find the value of the game:

	PLAYER B			
PLAYER A	−6	−1	4	3
	7	−2	−5	7

[(CO1, CO2, CO3, CO4)(Understand/LOCQ)]

7 + 5 = 12

Group – D

6. (a) Solve the following non-linear programming problem by the method of Lagrange multiplier:
Minimize $z = x_1^2 + (x_2 + 1)^2 + (x_3 - 1)^2$
subject to the constraint

$$x_1 + 5x_2 - 3x_3 = 6$$
$$x_1, x_2, x_3 \geq 0.$$

[(CO6)(Evaluate/HOCQ)]

(b) Consider the function $f(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 - 2x_3 - 7x_1 + 12$. Find the local optima point, if any, of the function. Determine also whether the local optima are global or not. [(CO6) (Understand/LOCQ)]

8 + 4 = 12

7. Use Kuhn-Tucker conditions to solve the following non-linear programming problem:
Maximize $z = 7x_1^2 + 6x_1 + 5x_2^2$
subject to the constraints

$$x_1 + 2x_2 \leq 10$$
$$x_1 - 3x_2 \leq 9$$
$$x_1, x_2 \geq 0$$

[(CO6)(Evaluate/HOCQ)]

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Group – E

8. Use Fibonacci Search method to minimize $f(x) = x^4 - 14x^3 + 60x^2 - 70x$ over $[0, 2]$ taking the tolerance limit 0.3 (consider $\varepsilon = 0.001$). [(CO1, CO5)(Apply/IOCQ)]

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9. Find the value of x in the interval $[0, 1]$ which minimizes the function $f(x) = x^2(x - 2.5)$ to within ± 0.05 of the exact value using Dichotomous search algorithm taking $\varepsilon = 0.001$.

[(CO1, CO5)(Apply/IOCQ)]

Cognition Level	LOCQ	IOCQ	HOCQ
Percentage distribution	14.58%	44.79%	40.62%

Course Outcome (CO):

After the completion of the course students will be able to

MATH4121.1 Describe the way of writing mathematical model for real-world optimization problems.

MATH4121.2 Identify Linear Programming Problems and their solution techniques.

MATH4121.3 Categorize Transportation and Assignment problems.

MATH4121.4 Apply the way in which Game Theoretic Models can be useful to a variety of real-world scenarios in economics and in other areas.

MATH4121.5 Apply various optimization methods for solving realistic engineering problems and compare their accuracy and efficiency.

MATH4121.6 Convert practical situations into non-linear programming problems and solve unconstrained and constrained programming problems using analytical techniques.

*LOCQ: Lower Order Cognitive Question; IOCQ: Intermediate Order Cognitive Question; HOCQ: Higher Order Cognitive Question

Department & Section	**Submission link:**
CSE	https://classroom.google.com/c/NDA0Nzc4NzE5MTc0/a/NDY0MzEwMTU2MzUx/details
ECE	https://classroom.google.com/c/NDA0Nzc4NzE5MTk1/a/NDY0MzEwMTU3MTAz/details