DIGITAL SIGNAL PROCESSING
(ELEC 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The fundamental period (N) of the signal $x(n) = 2cos^2(0.5\pi n)$ is
(a) $N = 2$ (b) $N = 4$
(c) $N = 1$ (d) $N = 0$

(ii) Sampling rate (T) of a given signal $x(n)$ can be changed by a non-integer factor using one of the following sequences of operations.
(a) Down sampling by L, filter and up sampling by L
(b) Up sampling by L, filter and down sampling by M
(c) Up-sampling, delay, and down-sampling
(d) $x(n)$ advanced by α and then time reversal

(iii) The time index interval of non-zero values of output $y(n)$ $(y(n) = x(n) * h(n))$ for the convolution between the sequences $x(n) = (3,\ 2,\ 4),\ -1 \leq n \leq 1$; and $h(n) = (-2,\ 3),\ 0 \leq n \leq 1$ is
(a) [0, 2] (b) [0, 5]
(c) [-1, 2] (d) [-1, 3]

(iv) The $z-$transform of a 'two-sided' signal $(x(n))$ $X(z) = \frac{z}{z+2} + \frac{z}{z-1}$ converges if $R.O.C\ is$
(a) $1 < |z| < \infty$ (b) $|z| > 2$
(c) $|z| < 2$ (d) $1 < |z| < 2$

(v) The z-transform of a 'two-sided' signal $(x(n))$ $X(z) = \frac{2z}{(z-1.5)(z+0.5)}$ with $R.O.C:\ 0.5 < |z| < 1.5$ is stable if
(a) $R.O.C$ to include the unit circle
(b) $R.O.C$ to exclude the unit circle
(c) the region $|z| < 1.5$ to include the unit circle
(d) the region $|z| < 0.5$ to excludes the unit circle

(vi) The DFT coefficient $X(1)$ of the four point segment $x(0) = 1,\ x(1) = 0,\ x(2) = 0\ ,\ x(3) = 1$ of a sequence $x(n)$ is
(a) $X(1) = 0$ (b) $X(1) = 1 + j$
(c) $X(1) = 1 - j$ (d) $X(1) = 1 + j2$

(vii) If the DFT of 4-point sequence of signal $x(n)$ is $X(k) = \{4, -j2\ ,\ 0,\ \ j2\};\ for\ k = 0\ to\ 3$, then DFT of $g(n) = x(n-2)$ is
(a) $G(k) = \{1,\ j4\ ,\ 2,\ \ -j4\};\ for\ n = 0\ to\ 3$
(b) $G(k) = \{0,\ \ j2\ ,\ 4,\ \ -j2\};\ for\ \ n = 0\ to\ 3$
(c) $G(k) = \{4,\ j2\ ,\ 0,\ \ -j2\};\ for\ n = 0\ to\ 3$
(d) $G(k) = \{1,\ -j\ ,\ 4,\ -j\};\ \ for\ n = 0\ to\ 3$

(viii) Let $x(n) = \{1,\ 2,\ \ 0,\ \ 3\}$ for $n = 0\ to\ 3$. The circularly shifted signal $x(n-3)$ is
(a) $\{0,\ 3,\ 1,\ 2\}$; for $n = 0\ to\ 3$ (b) $\{1,\ 3,\ 0,\ 2\}$; for $n = 0\ to\ 3$
(c) $\{3,\ 0,\ 1,\ 2\}$; for $n = 0\ to\ 3$ (d) $\{2,\ 0,\ 3,\ 1\}$; for $n = 0\ to\ 3$.

(ix) If $h(n) = \{2,\ 1,\ \ h(2), -1, -2\},\ for\ n = 0\ \ to\ \ 4$ is the impulse response of a linear-phase FIR filter, then the values of delay α (delay in the response of the filter) and $h(2)$ are
(a) 2.5, -1 (b) 2.5 , 1
(c) 2, 0 (d) 1.5, 0

(x) If a causal and stable discrete time system $H(z) = \frac{z}{z+0.2}$ is excited with a sinusoidal input $x(n) = 2\cos(0.05\pi n)\, u(n)$ then the amplitude (B) of the sinusoidal output response $y_{ss}(n) = B cos(0.05\pi n + \theta)$ at steady- state is
(a) 1.5 (b) 1.67
(c) 4.0 (d) 1.25

Group – B

2. (a) Explain the basic function of each component of a digital signal processing (DSP) unit with a block diagram. Show the nature of the signal at the output of each block while the input to the $'DSP'$ unit is $x(t) = \sin(t)\, u(t)$.

(b) A non-recursive filter of length $(M + 1)$ is described as $y(n) = b_o x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots \ldots + b_M x(n-M)$. Show that the impulse response sequence or samples are the filter 'tap-coefficients' (i.e. $b_i = 0,\ 1,\ 2, \ldots \ldots .M$).

(c) Is the given signal $x(n) = 1 + \cos(0.4\pi n) + \sin(0.5\pi n + 30^0)$ periodic? If so, determine the common period (N) of $x(n)$.

6 + 3 + 3 = 12

3. (a) A linear time invariant system with impulse response $h(n)$ is stable, in the bounded-input bounded-output sense, if and only if the impulse response is absolutely summable, that is, if $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

(b) The impulse response of a given discrete time system

$$h(n) = \left[-6\left(\frac{1}{2}\right)^n + 8\left(\frac{1}{2}\right)^n\right]u(n)$$

Check the causality of the system. Is the impulse response is absolutely summable?

(c) Realize the following digital filter using "direct-form-II" structure

$$\mathrm{H(z)} = \frac{(0.194-0.194z^{-1})}{(1+0.613z^{-1})}.$$

5 + 3 + 4 = 12

Group – C

4. (a) If $Z[x_1(n)] = X_1(z)$ with R.O.C: R_{x_1}; and $Z[x_2(n)] = X_2(z)$ with R.O.C: R_{x_2}; then for any scalars $a_1,\ a_2$ show that $Z[a_1x_1(n) + a_2x_2(n)] = a_1X_1(\mathrm{z}) + a_2X_2(\mathrm{z})$; R.O.C : $R_{x_1} \cap R_{x_1}$.

(b) Find the impulse response of the causal discrete transfer function $G(z) = \frac{z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{3}\right)}$ using the partial fraction method.

6 + 6 = 12

5. (a) An IIR filter is described as $y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = x(n)$ with zero initial conditions. Obtain the transfer function $H(z)$ of the IIR filter and hence obtain the output response of the system for $x(n) = 4u(n)$ and also get the dc gain of this filter.

(b) Find the impulse response of $H(z) = \frac{z(z+1)}{(z-0.4)(z+0.5)}$, and its ROC. Assume $h(n)$ is left –sided signal.

7+ 5 = 12

Group – D

6. (a) Define DFT of a finite sequence. Show that the $N-point\ DFT$ of a finite sequence signal $x(n)$ is periodic with period $'N'$.

(b) Four consecutive voltages $x(n) = \{0,\ 1,\ 2\ ,3\}$ for $0 \le n \le 3$, are recorded at time interval T (sec.). Evaluate DFT of the signal $(x(n))$, i.e. $X(k),\ for\ k = 0\ to\ 3$. Assume that $f_s = 8\ kHz$. Sketches the amplitude spectrum, phase spectrum and power spectrum of the signal $x(n)$.

4 + 8 = 12

7. (a) Discuss two basic properties of the DFT that lead to an efficient FFT algorithm? Explain the computational cost involved in computing an $N-point\ DFT$ using $DIF-FFT$ algorithm.

(b) The first four voltage values of a signal $x(n) = (0.5,\ 1,\ 1,\ 0.5)$, for $n = 0,\ 1,\ 2,\ 3$, are sampled at 125 Hz. Compute, DFT of the signal using direct calculation of $DIF-FFT$ algorithm or via signal flow graph.

5 + 7 = 12

Group – E

8. (a) Determine the frequency response of the causal discrete-time system $\boldsymbol{H}(\boldsymbol{z}) = \frac{\boldsymbol{z+1}}{\boldsymbol{z-0.707}}$ at digital frequencies $\Omega = 0,\ \frac{\pi}{4},\ \frac{\pi}{2},\frac{3\pi}{4},\ \pi.$ Sketch the Amplitude-vs- frequency and Phase-angle-vs- frequency.

(b) Drive the conditions for a realizable digital filter to have a linear phase characteristic, and discuss the advantages of such filter characteristic.

6 + 6 = 12

9. (a) A **high-pass *FIR*** filter having the following frequency specifications:
Pass-band edge frequency $\Omega_p = 0.5\pi\, rad/s$; stop-band edge frequency $\Omega_s = 0.125\pi\, rad/s$; cutoff frequency $\Omega_c = 0.312\pi\, rad/s$; transition band width $= 0.375\pi\, rad/s$; pass-band ripple $\delta_p \le 0.0575$; stop-band $\delta_s \le 0.001$. Sketch the tolerance diagram for the high-pass-filter indicating all design specifications.

(b) Transform the given analog filter $H_c(s) = \frac{5(s+2)}{(s+3)(s+4)}$ into a digital filter $H(z)$ using Bilinear Z-transform method with $T = 2sec$. Is the digital FIR filter stable and linear phase?

4 + 8 = 12

Department & Section	Submission Link
EE	https://classroom.google.com/c/MjI2MjE5NDQ2MDMy/a/MzY0MzE0Njc3MzI0/details