B.Tech/AEIE/ECE/IT/3rd Sem/MATH-2002/2015

2015

NUMERICAL & STATISTICAL METHODS

(MATH 2002)

Time Alloted : 3 Hours   **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and <u>any 5 (five)</u>
from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words
as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following :   **[10×1=10]**

i) The rate of convergence of bisection method is

(a) Linear   (b) quadratic

(c) cubic   (d) none of these

ii) Newton-Raphson method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is

(a) $x_{n+1} = \dfrac{x_n}{2}$ (b) $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{R}{x_n}\right)$

(c) $x_{n+1} = \dfrac{1}{2}\left(x_n - \dfrac{R}{x_n}\right)$ (d) None of these

iii) Simpson's 1/3 rule requries the

(a) even number of ordinates

(b) odd number of ordinates

(c) even or odd number of ordinates

(d) None of these

iv) The Gauss-Elimination method falls when any one of the pivotal element is

(a) 2   (b) 1

(c) −1   (d) 0

v) Which of the following is an iterative method for solving system of linear equations

(a) Gauss Elimination   (b) Gauss Siedel

(c) LU – factorization   (d) Newton Raphson

vi) If f(3) = 5 and f(5) = 3, then the linear interpolation function f(x) is

(a) 8 + x   (b) 8 − x

(c) 18 + x   (d) $8 + x^2$

vii) In Lagrange's polynomial the sum of the coefficients of $y_0, y_1,...,y_n$ i.e. the sum of the Lagrangian coefficients is

(a) 0   (b) 1

(c) 2   (d) 3

viii) A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads?

(a) $\left(\dfrac{1}{2}\right)^2$ (b) $^{10}C_2\left(\dfrac{1}{2}\right)^2$

(c) $\left(\dfrac{1}{2}\right)^{10}$ (d) $^{10}C_2\left(\dfrac{1}{2}\right)^{10}$

ix) The standard deviation of a uniformly distributed random variable between 0 and 1 is

(a) $1/\sqrt{12}$

(b) $5/\sqrt{12}$

(c) $1/\sqrt{3}$

(d) $7/\sqrt{12}$

x) The mean of Binomial distribution $B\left(10, \frac{2}{5}\right)$ is

(a) 4

(b) 6

(c) 5

(d) 0

GROUP - B

2. (a) Explain the geometrical interpretation of Newton-Raphson method. Use this method to evaluate the smallest positive root of the equation $x = \sin x - 0.25$ correct to three decimal places.

(b) Solve the following system of linear equations, if possible, by Gauss-Siedel iterative method correct to two decimal places.

$$2x + 10y - z = 13$$
$$10x + y - z = 12$$
$$2x + 2y - 10z = 14 \qquad \textbf{(2+4)+(6) = 12}$$

3. (a) Solve the following system of equation by using Gauss Elimination method

$$6x - y - z = 19$$
$$3x + 4y + z = 26$$
$$x + 2y + 6z = 22$$

(b) Find the real root of the equation $2x - \cos x - 3 = 0$ by bisection method correct up to two places of decimal.

(6)+(6) = 12

GROUP - C

4. (a) Find from the following table, the area bounded by the curve f(x) and the x-axis between x = 7.47 and x = 7.52

x	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

(b) Calculate value of y(102) from the following table. Correct your answer to two decimal places.

x	93.0	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

6+6 = 12

5. (a) Find the interpolating polynomial by suitable interpolation for the following data.

x	0	1	4	5
f(x)	−1	7	26	124

(b) Given $\frac{dy}{dx} = (y - x)$, where $y(0) = 2$. Use Runge-Kutta method to find the value of y(0.1) and y(0.2) correct to four decimal places taking h = 0.1

6+6 = 12

Group - D

6. (a) If A and B are independent events, then show that the following pairs are independent.

(i) A and $\overline{B}$

(ii) $\overline{A}$ and B

(iii) $\overline{A}$ and $\overline{B}$

(b) There are two identical urns containing 4 white and 3 red balls and 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find

(i) The probability that the ball drawn is white.

(ii) If the ball drawn is white, what is the probability that it is drawn from the first urn?   6+6 = 12

7. (a) The demand of a new product of a company is assumed to be a random variable with probability density function

$$f(x) = \begin{cases} xe^{-x^2/2} & , x \geq 0 \\ 0 & , \text{elsewhere} \end{cases}$$

Find (i) Expected demand quantity and (ii) probability that the demand exceeds 1.

(b) A and B throw alternatively with a pair of balanced dice. A wins if he throws a sum of six points before B throws a sum of seven points, while B wins if he throws seven points before A throws a sum of six points. If A begins the game, find the probability of his wining.

(3+3)+(6) = 12

GROUP - E

8. (a) A car-hire firm has two cars which it hires out day by day. The number of demand for a car on each day is distributed as a Poisson distribution, with the average number of demand per day 1.5. Calculate the proportion of days on which some demand is refused. [Given that $e^{-1.5} = 0.2231$].

(b) In a normal distribution 41% of the items are below 55 and 7% are above 64. Find the mean and standard deviation.

[Given P(0 < Z <1.34) = 0.43, P(−0.22 < Z < 0) = 0.09]

6+6 = 12

9. (a) Marks obtained by 10 students in the college test (X) and the university test (Y) are as follows :

X	41	45	50	68	47	77	90	100	80	100
Y	60	63	60	48	85	56	53	91	74	98

Apply the method of regression to estimate the marks a student could have obtained in the university test if he obtained 60 in the college test but was ill at the time of the university test.

(b) If the median and mode of the following frequency distribution are known to be 27 and 26 respectively, then find the values of a and b.

Values :	0-10	10-20	20-30	30-40	40-50
Frequency :	3	a	20	12	b

7+5 = 12