B.Tech/AEIE/CE/ECE/EE/3rd Sem/MATH-2001/2015

2015

MATHEMATICAL METHODS

(MATH 2001)

Time Alloted : 3 Hours   ***Full Marks : 70***

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for the following : **[10×1=10]**

i) The order of the pole of the function $\frac{\sin z}{z^3}$ is

(a) 1   (b) 2

(c) 3   (d) 4

ii) The value of the integral $\int_C \frac{dz}{z^2 - 2z}$, (where the circle $C : |z-2| = 1$ is traversed in the counter clockwise sense), is

(a) $-\pi i$ (b) $2\pi i$

(c) πi (d) none of these

iii) If $f(x) = x \sin x, -\pi \le x \le \pi$ be expanded in Fourier series as $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$, then a_1 =

(a) 0   (b) 1

(c) 1/2   (d) −1/2

iv) The Fourier sine transform of $e^{-|x|}$ is

(a) $\frac{1}{1+s^2}$ (b) $\frac{e^{-s}}{1+s^2}$

(c) $\frac{e^{s}}{1+s^2}$ (d) $\frac{s}{1+s^2}$

v) The general solution of the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (\lambda^2x^2 - p^2)y = 0$, where p is not an integer, is

(a) $c_1J_p(x) + c_2J_{-p}(x)$ (b) $c_1J_p\left(\frac{x}{\lambda}\right) + c_2J_{-p}\left(\frac{x}{\lambda}\right)$

(c) $c_1J_p(\lambda x) + c_2J_{-p}(\lambda x)$ (d) $c_1J_p\left(\frac{\lambda}{x}\right) + c_2J_{-p}\left(\frac{\lambda}{x}\right)$

vi) The partial differential equation of all spheres whose centres lie on the z – axis is

(a) $xq + yp = 0$ (b) $xq - yp = 0$

(c) $xq + \frac{1}{yp} = 0$ (d) $\frac{1}{xq} + \frac{1}{yp} = 0$

[Here, $p = \frac{\partial z}{\partial x}$; $q = \frac{\partial z}{\partial y}$]

vii) Partial differential equation of the surface $z = ax^2 - by^2$ is given by

(a) $2z = px - qy$ (b) $2z = px + qy$

(c) $z = px + qy$ (d) $z = ap^2 - bq^2$

viii) x^2 in terms of Legendre's polynomials is

(a) $\frac{P_2(x) - P_0(x)}{3}$ (b) $\frac{P_2(x) + P_0(x)}{3}$

(c) $\frac{2P_2(x) - P_0(x)}{3}$ (d) $\frac{2P_2(x) + P_0(x)}{3}$

ix) The period of the function $f(x) = |\sin x|$ is

(a) 2π (b) $\frac{\pi}{2}$

(c) 3π (d) π

x) If a function f is analytic throughout a simply connected domain D, then for every closed contour C lying in D

(a) $\int_C f(z)dz = 2\pi i$ (b) $\int_C f(z)dz = \pi i$

(c) $\int_C f(z)dz = \frac{\pi}{2} i$ (d) $\int_C f(z)dz = 0$

GROUP - B

2. (a) Prove that the function f(z) defined by $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(0) = 0$ is continuous and CR equations are satisfied at the origin, yet $f'(0)$ does not exist.

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(b) Evaluate $\oint_C \frac{e^z dz}{z(1-z)^3}$ where C is $|z - 1| = \frac{1}{2}$.

8+4=12

3. (a) Let C denote the boundary of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$, where C is described in the positive sense.

Use the Residue theorem to evaluate the integral

$$\int_C \frac{\cos z}{z\left(z^2 + 3/2\right)} dz$$

(b) Consider f to be an analytic function and its real part to be

$$u(x,y) = \frac{y}{x^2 + y^2}$$

(i) Find the conjugate harmonic of u.

(ii) Express f(z) in terms of z. **6+6=12**

GROUP - C

4. (a) Expand $f(x) = x^2$ in Fourier series in the interval $-\pi < x < \pi$ and deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots\ldots\ldots\ldots\ldots\ldots$$

(b) Expand the following function in a Fourier srries in $[-\pi, \pi]$

$$f(x) = \begin{cases} -\frac{1}{2}(\pi + x), & \text{when } -\pi \le x < 0 \\ \frac{1}{2}(\pi - x), & \text{when } 0 \le x \le \pi \end{cases}$$

5+7=12

5. (a) Find the Fourier transform of e^{-ax^2}, where $a > 0$

(b) Show that if $\hat{f}(w)$ is the Fourier transform of $f(x)$, then $\hat{f}(w-a)$ is the Fourier transform of $e^{iax} f(x)$

(c) Show $f * g = g * f$ where '*' indicates convolution.

6+4+2 = 12

Group - D

6. (a) Find a power series solution in power of x of the following differential equation :

$$(1-x^2)y'' - 2xy' + 2y = 0$$

(b) Using Rodrigue's formula prove the recurrence relation $nP_n = xP_n' - P_{n-1}'$ [where $P_n(x)$ denotes the Legendre's polynomial]

7+5 = 12

7. (a) Establish $\frac{d}{dx}\{J_n(x)\} = \frac{n}{x}J_n(x) - J_{n-1}(x)$ and hence show that $x^n J_n(x)$ is a solution of

$$x\frac{d^2y}{dx^2} + (1-2x)\frac{dy}{dx} + xy = 0$$

(b) Solve, by the finite difference method, the boundary value problem $x^2y''(x) - 2y(x) + x = 0$, $2 < x < 3$, where $y(2) = 0$, $y(3) = 0$, taking $h = \frac{1}{4}$.

5+7 = 12

GROUP - E

8. (a) Verify that $u(x,y) = a\ln(x^2 + y^2) + b$ satisfies Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and determine a and b so that u satisfies the boundary conditions $u = 0$ on the circle $x^2 + y^2 = 1$ and $u = 3$ on the circle $x^2 + y^2 = 4$.

(b) Solve by the method of separation of variables : $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, $x > 0$, $t > 0$, where $u(x, 0) = 6e^{-3x}$

6+6 = 12

9. (a) Apply Charpit's method to find the complete integral of $2(u + xp + yq) = yp^2$

where $p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial y}$

(b) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$, $0 < x < l$, $t > 0$

given that $u(0,t) = u(l,t) = 0$, $u(x,0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$ where $f(x)$ is a known function.

6+6 = 12