COMPUTATIONAL MATHEMATICS
(MATH 3221)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and
any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The generating function of the sequence $\{1, 3, 9, 27, 81, 243, \ldots., 3^n, \ldots.\}$ is
(a) $\frac{1}{1-x^3}$ (b) $\frac{1}{1+x^3}$ (c) $\frac{1}{1+3x}$ (d) $\frac{1}{1-3x}$

(ii) The third harmonic number $H_3 =$
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{11}{6}$

(iii) The second Bernoulli number $B_2 =$
(a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) 0

(iv) Which of the following is a Fibonacci number?
(a) 235 (b) 144 (c) 133 (d) 35

(v) $\varphi(32) =$
(a) 12 (b) 8 (c) 31 (d) 16

(vi) $\left[\lfloor\sqrt[4]{3}\rfloor + \sqrt[4]{3}\right] =$
(a) 1 (b) 2 (c) 3 (d) 4

(vii) The remainder in the division of $10^{100} - 1$ by 99 is
(a) 1 (b) 9 (c) 0 (d) 10

(viii) $\Delta(x^{\underline{10}})$ is a polynomial in x whose degree is
(a) 10 (b) 9 (c) 11 (d) 8

(ix) The generating function of the sequence $\{1, -3, 9, -27, 81, \ldots, (-1)^n 3^n, \ldots\}$ is
(a) $\frac{1}{1-3x}$ (b) $\frac{1}{(1+3x)^2}$ (c) $\frac{1}{1+3x}$ (d) $\frac{1}{1+x^3}$

(x) Which one of the following is a Fibonacci number?
(a) 89 (b) 235 (c) 146 (d) 36.

Group – B

2. (a) Express $S_n = \sum_{k=1}^{n} k2^k$ as a function of n. Show your work in detail.

(b) Consider the Tower of Hanoi problem. Let T_n denote the maximum number of moves that will transfer n disks from one peg to another, if only one disk is moved at a time, and a larger one is never moved onto a smaller one. Prove that $T_n \geq 2T_{n-1} + 1$ for $n > 0$.

6 + 6 = 12

3. (a) Prove that $\sum_{k=0}^{n} k^2 = \frac{n(n+1)(n+2)}{6}$ by the perturbation method.

(b) Prove that $\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x^{\underline{n}}}{(x-m)^{\underline{n}}}$.

7 + 5 = 12

Group – C

4. (a) Prove the recurrence relation for the Sterling numbers of the first kind, $\begin{bmatrix} n \\ k \end{bmatrix}$:
$$\begin{bmatrix} n \\ k \end{bmatrix} = (n-1)\begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix}$$
where n, k are positive integers and $2 \leq k \leq n$.

(b) The generating function of the Bernoulli numbers B_n is $\frac{x}{e^x-1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$.
Use this to prove that for any integer$k \geq 1$, the following recurrence relation hold true.
$$\frac{B_k}{1!k!} + \frac{B_{k-1}}{2!(k-1)!} + \frac{B_{k-2}}{3!(k-2)!} + \frac{B_{k-3}}{4!(k-3)!} + \dots\dots + \frac{B_{k-2}}{n!1!} + \frac{1}{(n+1)!} = 0.$$

6 + 6 = 12

5. (a) LetF_k denote the Fibonacci numbers. Let $m \geq 2$ be a fixed integer. Prove that $F_{m+n} = F_{m-1}F_n + F_m F_{n+1}$ for all integers $n \geq 1$, by induction on n.

(b) Let r, k be positive integers and $r > k$. Prove that
$$(r-k)\binom{r}{k} = r\binom{r-1}{k}$$

6 + 6 = 12

Group – D

6. (a) Show that $18! + 3^{310} + 4^{416} + 20! \equiv 1 (mod\ 19)$. Show your calculations and state every theorem that you use.

(b) (i) Let n be a positive integer. Prove that if $(n-1)! \equiv -1\ (mod\ n)$, then n must be a prime number.

(ii) Let a, b, m, k be positive integers. If $a \equiv b\ (mod\ m)$, prove that $a^k \equiv b^k\ (mod\ m)$.

6 + (3 + 3) = 12

7. (a) Given that a has order 3 modulo p, where p is an odd prime, show that $(a+1)^6 \equiv 1 \ (mod \ p)$.

(b) Let n, m be positive integers such that $n > m$.
Prove the Dirichlet Box Principle: If n objects are put into m boxes, then some box must contain $\geq \left\lceil \frac{n}{m} \right\rceil$.

6 + 6 = 12

Group – E

8. (a) Solve the recurrence relation $a_{n+1} - a_n = 3n^2 - n; n \geq 0, a_0 = 3$.

(b) Find a formula for the general term F_n of the Fibonacci sequence $0, 1, 1, 2, 3, 5, 8, 13, \ldots\ldots$

6 + 6= 12

9. (a) Solve the recurrence relation by using the method of generating function $a_{n+1} + 4a_n + 4a_{n-1} = n-1; n \geq 1$, given $a_0 = 0$ & $a_1 = 1$.

(b) Use mathematical induction to prove that $5^{2n} - 2^{5n}$ is divisible by 7.

6 + 6 = 12

Department & Section	Submission Link
CSE	https://classroom.google.com/c/MzExNTQ4NTg0MTY3/a/MzQ0MTcyNTUzNjY5/details