PROBABILITY & NUMERICAL METHODS
(MATH 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Which of the following relation is correct?
(a) $\Delta^n x^n = (n+1)!$ (b) $\Delta^n x^n = n!$
(c) $\Delta^n x^n = 0$ (d) $\Delta^n x^n = n$

(ii) If $\frac{dy}{dx} = x + y$ and $y(1) = 0$, then $y(1.1)$ by Euler's method is $[h = 0.1]$:
(a) 0.1 (b) 0.3 (c) 0.5 (d) 0.9.

(iii) Condition for convergence of Newton- Raphson method is:
(a) $|f(x)\, f'(x)| < \{f''(x)\}^2$ (b) $|f(x)\, f''(x)| < \{f'(x)\}^2$
(c) $|f(x)\, f'(x)| > \{f''(x)\}^2$ (d) $|f(x)\, f''(x)| > \{f'(x)\}^2$

(iv) A random variable X is uniformly distributed in the interval $[a, b]$. Then the mean of X is:
(a) $\frac{1}{b-a}$ (b) $\frac{a+b}{2}$ (c) $\frac{b-1}{a-1}$ (d) $\frac{b}{a}$

(v) An event A is independent of itself, then $P(A)$ is:
(a) 1 (b) 0.5 (c) 0.25 (d) 0.75.

(vi) The probability of obtaining the sum as 10, when two dice are thrown is:
(a) 3/36 (b) 2/36 (c) 1/36 (d) 5/36.

(vii) The probability of obtaining a question from NUMERICAL METHODS is 3/7 and the probability of obtaining a question from PROBABILITY is 4/7. Considering that the events are independent, then the probability of obtaining at least one question from the topics is:
(a) 39/49 (b) 0.5 (c) 37/49 (d) 0.

(viii) A random number x is drawn from the set $\{2, 3, 4, 5, \dots, 50\}$. The probability that x is prime is

(a) 0 (b) $\frac{15}{49}$ (c) $\frac{14}{49}$ (d) $\frac{17}{49}$.

(ix) The function $f(x,y) = \begin{cases} kxy(x+y), & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0, & elsewhere \end{cases}$ can be a p.d.f. if the value of k is:

(a) 3 (b) -3 (c) 1/3 (d) -1/3.

(x) The degree of precision of Simpson's one-third rule is:

(a) 1 (b) 2 (c) 3 (d) 5.

Group – B

2. (a) Use Newton-Rapshson Method to solve the following equation $x + log_e x - 2 = 0$, correct to 4 significant figures.

(b) Solve the following system of linear equations by using Gauss Elimination method:

$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$

6 + 6 = 12

3. (a) Compute $f(0.29)$, from the following table, by using Newton Backward interpolation formula, correct up to three decimal places:

x	0.20	0.22	0.24	0.26	0.28	0.30
$f(x)$	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

(b) Evaluate $\int_0^1 \sqrt{1-x^2}dx$ using Simpson's 1/3 rule for $n = 6$ correct up to five decimal places.

7 + 5 = 12

Group – C

4. (a) There are two identical urns containing 4 white and 3 red balls; 3 white and 7 red balls. An urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. What is the probability that it is from the first urn, given that the ball drawn is white?

(b) Two unbiased dice are thrown. Find the conditional probability that two fives occur if it is known that the total sum is divisible by 5?

(c) Prove that if A and B are independent events then A^c and B^c are also independent.

5 + 4 + 3 = 12

5. (a) When a computer goes down, there is a 75% chance that it is due to an overload and a 15% chance that it is due to a software problem. There is an 85% chance that it is due to an overload or software problem. What is the probability that both of these problems are at fault? What is the probability that there is a software problem but no overload?

   (b) Assume that 95% of all cryptographic messages are authentic. Furthermore, assume that only 0.1% of all unauthentic messages are sent using the correct key and that all authentic messages are sent using the correct key. Find the probability that a message is authentic given that the correct key is used.

   (c) Two dice are thrown. What is the probability that the product of the numbers appearing in the two faces is an even number?

4 + 5 + 3 = 12

Group – D

6. (a) With the usual notations, find p for a binomial distribution if $n = 6$ and if $9P(X = 4) = P(X = 2)$.

   (b) Consider that X is normally distributed and the mean of X is 12 and standard deviation is 4.
   (i) Find a, such that $P(X > a) = 0.24$.
   (ii) Find b, c, such that $P(b < X < c) = 0.5$ and $P(X > c) = 0.25$.

   (c) A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with average number of demand per day 1.5. Calculate the proportion of days on which neither car is used and the proportion of days on which some demand is refused.

3 + 6 + 3 = 12

7. (a) In four tosses of a coin, let X be the number of heads. Tabulate the 16 possible outcomes with the corresponding values of X. By simple counting, derive the probability mass function of X and hence calculate the expected value of X.

   (b) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible:
   Variance of $X = 9$
   Regression equation: $8X - 10Y + 66 = 0,\ 40X - 18Y = 214$
   What were
   (i) The mean values of X, Y.
   (ii) The standard deviation of Y.

6 + (3 + 3) = 12

Group – E

8. (a) A two dimensional random variables (X, Y) has probability mass function is given by $p_{xy} = P(X = x, Y = y) = k(2x + 3y), x = 0,1,2\ ; y = 1,2,3$, k is constant.

Find (i) the value of k
(ii) Find all Marginal and conditional p.m.f. of X and Y. Are they independent?
(iii) Also find the probability distribution of $X + Y$.

(b) The joint distribution of X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{2}, & \text{if } (x,y) \in \mathcal{R} \\ 0, & \text{otherwise} \end{cases}$$

Where $\mathcal{R}$ is the interior of the triangle of area 2 square units having vertices $(0,0),(2,0)$ and $(1,2)$. Find the marginal density function of X. Also find $P(X \leq 1, Y \leq 1)$.

6 + 6 = 12

9. (a) An urn always contains 2 balls. Ball colours are red and blue. At each stage a ball is randomly chosen and then replaced by a new ball, which with probability 0.8 is the same colour, and with probability 0.2 is the opposite colour, as the ball it replaces. Let X_n be the number of red balls in the urn after n^{th} selection. Then $X_n,\ n \geq 0$ forms a Markov chain. Find out the state space of this Markov chain. Also find the probability transition matrix.

(b) A man is at an integral point on the x-axis between the origin and the point 3. He takes a unit step to the right with probability $\frac{1}{3}$ or to the left with probability $\frac{2}{3}$, unless he is at the origin, where he takes a step to the right to reach the point 1 or is at the point 3, where he takes a step to the left to reach the point 2. What is the probability that (i) he is at the point 1 after 3 walks? and (ii) he is at the point 1 in the long run?

6 + 6 = 12

Note:

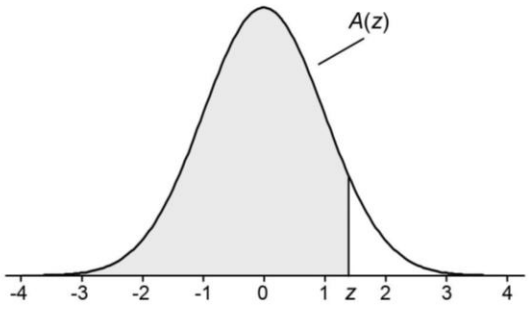
1. **Students having backlog in MATH2202 (CSE)(old syllabus) and if not joined in any Google classroom for this paper code yet, are advised to follow both Step-I and Step-II as mentioned below in order to submit the answer-scripts properly.**

2. **Students who have already joined any Google Classroom for MATH2202 (CSE)(old syllabus) can directly go to Step-II as mentioned below.**

Department & Section	Steps	Link
CSE (Backlog)	Step-I : Join Google Classroom using institutional email account	https://classroom.google.com/c/Mzc3MDUyNTcyODIz?cjc=kbpxsw7
	Step-II: Submit the answer script.	https://classroom.google.com/c/Mzc3MDUyNTcyODIz/a/MzE5MTM2OTI1OTk3/details

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							