MATHEMATICAL METHODS (MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Let $F(s)$ be the Fourier transform of $f(x)$, then the Fourier transform of $f(ax)$ is
(a) $\frac{1}{a}F\left(\frac{s}{a}\right)$ (b) $F\left(\frac{s}{a}\right)$ (c) $\frac{1}{a^2}F\left(\frac{s}{a}\right)$ (d) $\frac{1}{a}F(s)$.

(ii) For the differential equation $3xy'' + 2x(x-1)y' + 5y = 0$,
(a) $x = 0$ is a regular singular point (b) $x = 0$ is an irregular singular point
(c) $x = 0$ is an ordinary point (d) no singular point exist.

(iii) The solution of $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ is
(a) $z = f_1(y+x) + f_2(y-x)$ (b) $z = f_1(y+x) + f_1(y-x)$
(c) $z = f(x^2 - y^2)$ (d) $z = f(x^2 + y^2)$.

(iv) If $f(z) = \frac{\sin z}{z^4 - 2z^3}$, then $z = 0$ is pole of order
(a) 1 (b) 2 (c) 3 (d) 4.

(v) The value of m such that $3y - 5x^2 + my^2$ is a harmonic function is
(a) 5 (b) -5 (c) 0 (d) 3.

(vi) Particular integral of the partial differential equation $r - s + t = cos(2x + 3y)$ (where $r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial x \partial y}, t = \frac{\partial^2 z}{\partial y^2}$) is
(a) $-cos(2x + 3y)$ (b) $cos(2x + 3y)$
(c) $sin\ (2x + 3y)$ (d) $-sin\ (2x + 3y)$.

(vii) The order of partial differential equation $p \tan y + q\ tan\ x = \sec^2 z$ (where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$) is
(a) 1 (b) 2 (c) 0 (d) 3.

(viii) The value of $\int_{-1}^{1} P_0(x)\, dx$, where $P_n(x)$ is the Legendre polynomial of degree n is
(a) 0 (b) 1 (c) 2 (d) −1.

(ix) The period of the function $f(x) = |\sin x|$ is
(a) 2π (b) $\frac{\pi}{2}$ (c) 3π (d) π.

(x) Let $f(z) = u(x,y) + iv(x,y)$ be an analytic function, then $f^{'}(z)$ is
(a) $\frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ (b) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial x}$ (c) $\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial x}$ (d) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}$.

Group – B

2. (a) Prove that the function defined by

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is not analytic at the origin although Cauchy-Riemann equations are satisfied at the origin.

(b) Find the value of the integral $\int_0^{1+i} (x - y + ix^2)\, dz$ along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to the imaginary axis from $z = 1$ to $1 + i$.

6 + 6 = 12

3. (a) Evaluate $\int_c \frac{1}{(z^2+4)^3}\, dz$ where $c: |z - i| = 2$ using Cauchy residue theorem.

(b) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series in the regions *(i)* $|z| < 1$ *(ii)* $1 < |z| < 3$.

5 + 7 = 12

Group – C

4. (a) Find half-range sine series of the function $f(x) = x, 0 \leq x < \pi$ and hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(b) Using Parseval's identity for the function $f(x) = \begin{cases} -x, & -\pi \leq x < 0 \\ x, & 0 \leq x < \pi \end{cases}$, show that $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots \infty = \frac{\pi^4}{96}$.

6 + 6 = 12

5. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{\sin s - s\cos s}{s^3} \cos\frac{s}{2}\, ds$.

(b) Find the inverse Fourier transform of the function $\frac{1}{3-4is-s^2}$.

(6 + 2) + 4 = 12

Group – D

6. (a) Find the series solution of $y'' + xy = 0$ about the point $x = 0$.

   (b) Prove that $2nJ_n(x) = x\{J_{n-1}(x) + J_{n+1}(x)\}$.

7 + 5 = 12

7. (a) Find the value of $P_n(1)$. Hence prove that $P_n'(1) = \frac{1}{2}n(n+1)$.

   (b) Prove that $J_{-n}(x) = (-1)^n J_n(x)$, where n is any positive integer.

(3 + 3) + 6 = 12

Group – E

8. (a) Form the partial differential equation by eliminating the arbitrary function 'f' from $f(xy + z^2,\ x + y + z) = 0$.

   (b) Solve $p + q = x + y + z$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

6 + 6 = 12

9. (a) Solve the following partial differential equation by D-operator method $(D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y})$

$$\left(D^2 - DD' - 2D'^2\right)z = (y-1)e^x.$$

   (b) Solve $4xyz = pq + 2px^2y + 2qxy^2$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

6 + 6 = 12

Department & Section	Submission Link
CE-A	https://classroom.google.com/c/MzExNTQ3NjcwMjk3/a/MzA1NzAwMjkxNDIx/details
CE-B	https://classroom.google.com/c/MzA5NDU2NTczOTA3/a/MzcyODc3MTc1MzU5/details
EE	https://classroom.google.com/c/MzExNTQ3MzM1NDA3/a/MzczMDUyODIzNTUx/details