MATHEMATICS - II
(MATH 1201)

Time Allotted : 2 hrs. **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Let X be a random variable which follows normal distribution with mean zero and variance one. Then $E(|X|)$ is

(a) 0 (b) $\sqrt{\frac{2}{\pi}}$ (c) $\sqrt{\frac{1}{\pi}}$ (d) $\sqrt{\frac{1}{2\pi}}$.

(ii) A random variable X has the probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx & for\ 0 < x < 1 \\ 0 & otherwise \end{cases}$$

If the expected value $E(X) = \frac{2}{3}$, then $P(X < 0.5)$ is

(a) 0.5 (b) 1 (c) 0.25 (d) 0.

(iii) If the number 37.46235 rounded off to four significant digits then the percentage of error is

(a) 6.27×10^{-3} (b) 6.27×10^{-5}
(c) 6.27×10^{-2} (d) 6.27×10^{3}.

(iv) Let $f(x) = 0$ be an equation such that $f(a)\,f(b) > 0$ for two real numbers $a$and$b$, then which one of the following is true:

(a) at least one root of $f(x) = 0$ lies in (a, b)
(b) no root of $f(x) = 0$lies in (a, b)
(c) an odd number of roots of $f(x) = 0$lies in (a, b)
(d) either no root or an even number of roots of $f(x) = 0$ lies in (a, b).

(v) Let S be a sample space and two mutually exclusive events A and B be such that$A\ U\ B = S$. If $P(.)$ denotes the probability of the event, the maximum value of $P(A)P(B)$ is

(a) 1 (b) 0.5 (c) 0.25 (d) 0.

(vi) Let G be an undirected graph with n vertices and 25 edges such that each vertex of G has degree at least 3. Then the maximum possible value of n is
(a) 13 (b) 14 (c) 15 (d)16.

(vii) Which one of the following is TRUE for any simple connected undirected graph with more than 2 vertices?
(a) No two vertices have the same degree
(b) At least two vertices have the same degree
(c) At least three vertices have the same degree
(d) All vertices have the same degree.

(viii) A two-faced fair coin has its faces designated as head (H) and tail (T). This coin is tossed three times in succession to record the following outcomes: H, H, H. If the coin is tossed one more time,the probability (up to one decimal place) of obtaining H again, given the previous realizations of $H, H$and H, would be
(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$.

(ix) The value of $\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{1}{4}\right)$ is
(a) $\sqrt{2\pi}$ (b) $\sqrt{2}\,\pi$ (c) $2\sqrt{\pi}$ (d)$\frac{\pi}{\sqrt{2}}$.

(x) The Laplace transform of $f(t) = 2\sqrt{\frac{t}{\pi}}$ is $s^{-\frac{3}{2}}$, then the Laplace transform of $g(t) = \sqrt{\frac{1}{\pi t}}$is
(a) $s^{\frac{1}{2}}$ (b) $s^{\frac{3}{2}}$ (c) $3s^{-\frac{5}{2}}$ (d) $s^{-\frac{1}{2}}$.

Group – B

2. (a) Two bags contain 5 white, 3 black balls and 10 white, 3 black balls respectively. Two balls are transferred at random from first bag to second bag and then a ball is drawn at random from latter. The ball so drawn is found to be white in colour. Find the probability that the transferred balls were both white.

(b) Suppose that an airplane engine will fail, when in flight, with probability $1 - p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

6 + 6 = 12

3. (a) A bowl contains 10 chips of which 8 are marked Rs.200 each and 2 are marked Rs.500 each. Let a person draw 3 chips at random from the bowl without replacement. If the person is to receive the sum of resulting amounts marked on the drawn chips, find his expectation.

(b) Assuming that the height distribution of a group is normally, find the mean and standard deviation if 84% of the men have heights less than 65.2 inches and 68% have heights lying between 62.8 and 65.2 inches.

6 + 6 = 12

Group – C

4. (a) Solve the following system of equations by Gauss-Seidal iteration method correct to three significant figures:

$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$

(b) Find the iterative formula for finding $\frac{1}{\sqrt{N}}$ where N is a real number, using the Newton-Raphson formula. Hence find $\frac{1}{\sqrt{14}}$, correct up to 4 places of decimal.

7 + 5 = 12

5. (a) The current i in an electric circuit is given by $i = 10e^{-t}\ sin\ (2\pi t)$, where t is in seconds. Using Regula-Falsi method, find the minimum value of t correct to four decimal places for $i\ = 2\ amp$.

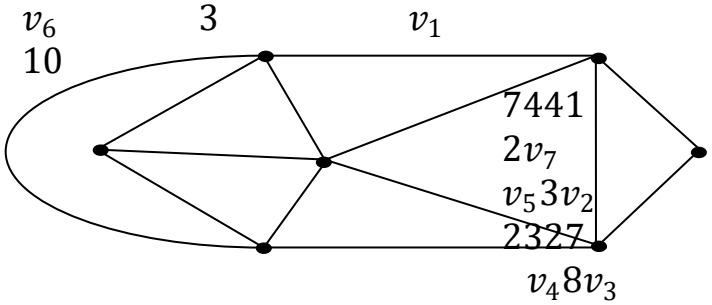
(b) Use Euler's modified method to find $y(0.4)$, for $\frac{dy}{dx} = x + \left|\sqrt{y}\right|$, $y(0) = 1$, by taking $h = 0.2$.

6 + 6 = 12

Group – D

6. (a) Prove that the number of pendant vertices in a binary tree is $\frac{n+1}{2}$, where n is the number of vertices in the tree.

(b) By Dijkstra's algorithm find the shortest path and the length of the shortest path from the vertex v_2 to v_5 in the following graph:



5 + 7 = 12

7. (a) If G be a bipartite graph with 22 vertices with partite sets U and V where U contains 12 vertices. Suppose every vertex of U has degree 3 while every vertex of V has degree either 2 or 4. How many vertices of G have degree 2?

(b) Does there exist a simple graph corresponding to the following incidence matrix? Justify your answer.

$$A(G) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) Examine if the following two graphs are isomorphic. Justify your answer.

5 + 3 + 4= 12

Group – E

8. (a) Evaluate $\int_0^\infty \frac{\sin^3 t}{t} dt$ by finding the Laplace transform of $F(t) = \frac{\sin^3 t}{t}$.

(b) Show that $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}} = \pi$.

7 + 5 = 12

9. (a) Find the inverse Laplace transform of $\frac{1}{(s^2+2s+5)^2}$ by convolution theorem.

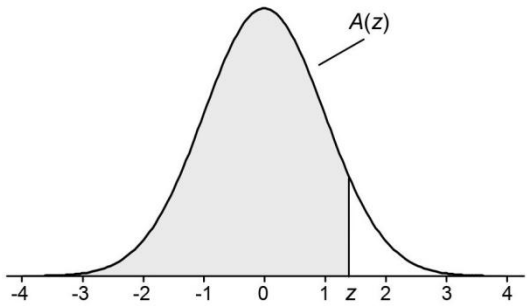
(b) Show that $\int_0^{\pi/2} \tan^p\theta \, d\theta = \frac{\pi}{2}\sec\frac{p\pi}{2}$, where $-1 < p < 1$.

7 + 5 = 12

Department & Section	Submission Link
AEIE	https://classroom.google.com/c/MzExNTY1NTMxNjY1/a/MzcyODg1ODE5OTIx/details
BT	https://classroom.google.com/c/MzA5NDU2NTczOTYx/a/MzcyODgxODg1MDcx/details
CHE	https://classroom.google.com/c/MzExNTgwNTA0Mzc0/a/MzcyODg0NTI5NDU5/details
CE-A	https://classroom.google.com/c/MzExNTQ4NTQ1MjQx/a/MzA1NzAwMTUwMzMz/details
CE-B	https://classroom.google.com/c/MzExNTUyNjgyNDA1/a/MzA1NzAwMTUwMzgx/details
CSBS	https://classroom.google.com/c/MzExNTUzNTExMTY3/a/MzczMDUxNTU5MDgw/details
CSE-A	https://classroom.google.com/c/MzExNjIyMzA3ODA4/a/MzcwNzYzODU4NTg5/details
CSE-B	https://classroom.google.com/c/MzExNjIyMzA3ODMy/a/MzcwNzYzODU4NjU1/details
CSE-C	https://classroom.google.com/c/MzA5NDU2NTczOTE2/a/MzcyODc3ODg1NDg3/details
ECE-A	https://classroom.google.com/c/MzExOTU5NjQ0Nzg4/a/MzcyNzU1OTc0OTgz/details
ECE-B	https://classroom.google.com/c/MzExOTU5NjQ0ODA0/a/MzcyNzU1OTc1MDI5/details
ECE-C	https://classroom.google.com/c/MzExNTQ3NjE2NDM3/a/MzcyODg1ODE5OTg1/details
EE	https://classroom.google.com/c/MzExNTQ1OTIzNjY2/a/MzA1Njk5NTgzMzEy/details
IT	https://classroom.google.com/c/MzExNTU0MzkyODQx/a/MzcwODQyNDI1Mzcy/details
ME-A	https://classroom.google.com/c/MzExNTY1NTMxNjky/a/MzcyODg1ODE5ODY3/details
ME-B	https://classroom.google.com/c/MzExNTgwNTA0MzUy/a/MzcyODg0NTI5NjYx/details

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							