FINITE ELEMENT METHOD
(MECH 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For deriving solution function from Governing equation using 'Principle of Total Stationary Potential (PSTP)' following method is used
(a) Double Integration Method (b) Area Moment Method
(c) Rayleigh Ritz Method (d) Navier Stokes Method.

(ii) Stiffness matrix of a BAR element in global coordinate system used for Truss is a:
(a) 3 × 3 matrix (b) 4 × 4 Matrix
(c) 2 × 2 matrix (d) Single column matrix.

(iii) Which one among the following is correct regarding stiffness matrix of a BEAM element?
(a) $\frac{AE}{3L}[4 \times 4]$ (b) $\frac{EI}{L}[4 \times 4]$
(c) $\frac{EI}{L^3}[4 \times 4]$ (d) $\frac{AE}{L^2}[4 \times 4]$
All the symbols and notations carry their usual meaning.

(iv) The elemental stiffness matrix of a BEAM element can be expressed as
(a) $\int [B]^T [B] E \, dv$ (b) $\int [B] [B]^T E \, dv$
(c) $\int [B] E \, dv$ (d) $\int [B] \, dv$
All the symbols and notations carry their usual meaning.

(v) The coefficient in stress-strain relation for a linear, elastic, isotropic material under plane strain condition is given by-

(a) $\frac{E}{(1+\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (b) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(c) $\frac{E}{(1-2\nu)}\begin{bmatrix} 1-\nu & 0 & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$ (d) $\frac{E}{(1+\nu)(1-2\nu)}\begin{bmatrix} \nu & 0 & 0 \\ \nu & \nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix}$

(vi) CST element possesses
(a) Constant field variable throughout the element.
(b) Derivative of the field variable is constant throughout the element.
(c) Variation of the field variable is quadratic throughout the element.
(d) Variation of the field variable is cubic throughout the element.

(vii) For a Four-Noded Rectangular element having length 'l' and width 'w', if one of the shape functions in user-coordinate (x, y) is expressed as $N_1 = \frac{x}{l}\left(1 - \frac{y}{w}\right)$ then the corresponding shape function in normalized coordinate (ξ, η) will be
(a) $\frac{1}{4}(1-\xi)(1+\eta)$ (b) $\frac{1}{4}(1+\xi)(1-\eta)$
(c) $\frac{1}{2}(1-\xi)(1-\eta)$ (d) $\frac{1}{4}(1-\xi)$

(viii) The weight factor W_i for a single point integration by Gauss Quadrature formula is
(a) 4 (b) 3 (c) 1 (d) 2.

(ix) Which of the following is not a numerical method?
(a) Finite Element Method (b) Finite Volume Method
(c) Rayleigh Ritz method (d) Finite Difference Method.

(x) Which one of the following software is not a FEA dedicated software?
(a) NASTRAN (b) MSC ADAMS (c) ANSYS (d) PTC Creo.

Group – B

2. For the following differential equation and stated boundary conditions, obtain a two-term solution using the Galerkin's method of weighted residuals using the specified trial functions. Compare the two-term solution to the exact solution.

$\frac{d^2y}{dx^2} + 5x^2 = 2, \qquad 0 \leq x \leq 1$

y(0) = 0 and y(1) = 0 (Boundary conditions)

$N_1(x) = x(x-1)$ *and* $N_2(x) = x^2(x-1)$ (Shape Function)

(6 + 6) = 12

3. A simply supported beam is imposed with a distributed load of 'q' as shown in figure 1. Write down the expression for total potential energy for this beam under this loading and boundary condition and then determine the solution function for beam deflection 'v' with respect to the domain variable 'x' considering the approximate solution $[v(x) \approx c_1 \sin(\pi x/L)]$ using Rayleigh-Ritz method. Here c_1 is a constant.

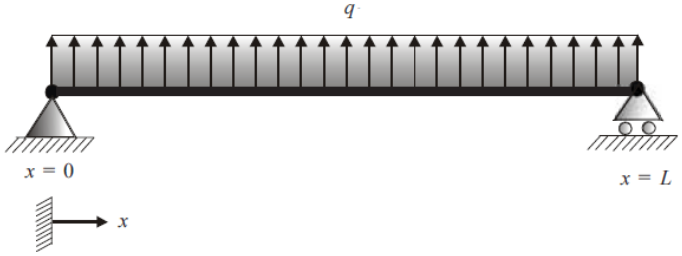

Fig.1

(2 + 10) = 12

Group – C

4. Fig.2 shows a two-member plane truss supported by a linearly elastic spring. The truss members are of a solid circular cross section having d = 20 mm and E = 80 GPa. The linear spring has stiffness constant 50 N/mm. Determine Global Stiffness matrix of the problem and displacement of the load point.

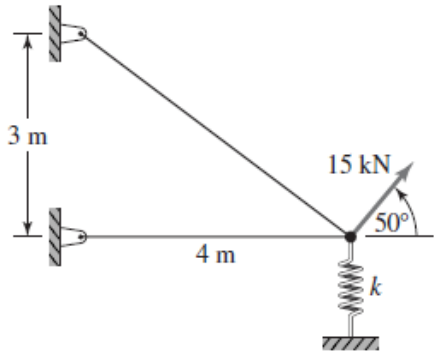

Fig.2

(7 + 5) = 12

5. Fig.3 below shows a beam imposed with point load and distributed load. Construct the required FEA formulation to find the vertical displacement of the beam at point B and midpoint of CD. Consider: E = 210MPa, I =1600 mm^4, l = 3m, F = 100 N and q = 200 N/m.

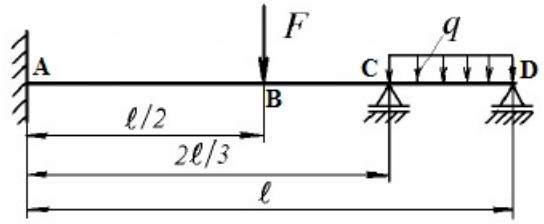

Fig.3

12

Group – D

6. Evaluate the given integral (i) analytically and (ii) using Gauss-Legendre formula.

$$\int_1^5 \int_0^6 (5y^3 + 2x^2 + 5)dxdy$$

(4 + 8) = 12

7. (a) What do you understand by CST element? Present a brief and to-the-point discussion on your understanding.

(b) Consider the isoparametric quadrilateral element in Fig.4. Map the point P($\xi = 0.3, \eta = 0.7$) in the parent element to the corresponding physical point in the quadrilateral element.

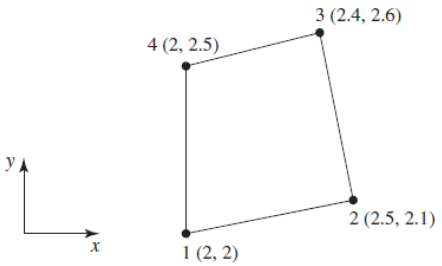

Fig.4

3 + 9 = 12

Group – E

8. Discuss in detail about the methodology to evaluate stress in a beam using Finite Element Method.

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9. Discuss in detail about Pre Processing, Solving and Post Processing steps followed by any FEA software.

(5 + 7) = 12

Department & Section	Submission Link
ME	https://classroom.google.com/c/Mjk3MjM4NTQ4NDk4/a/MzY0NjI4NDkwOTg1/details