B.TECH/ME/4TH SEM/MATH 2001 (BACKLOG)/2021

MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of $\oint_C \frac{dz}{z-4}$ where C is the circle $|z-1| = 2$ is

(a) $2\pi i$ (b) 0 (c) πi (d) $4\pi i$

(ii) Which of the following functions has an essential singularity at $z = 0$

(a) $\frac{1}{z}$ (b) $\frac{1}{z} + \frac{1}{z^2}$ (c) e^{z^2} (d) $e^{\frac{1}{z^3}}$

(iii) The period of $cos 2\pi x$ is

(a) 2π (b) 1 (c) 2 (d) none of these

(iv) The Fourier sine transform of $f(x) = e^{-ax}, a > 0$ is

(a) $\frac{s}{(a^2+s^2)}$ (b) $\frac{a}{(a^2+s^2)}$ (c) $\frac{2s}{(a^2+s^2)}$ (d) none of these

(v) $\int_{-1}^{1} P_0(x) dx =$

(a) 4 (b) 1 (c) 2 (d) 0

(vi) For the differential equation $x^2(1-x)y'' + xy' + y = 0$

(a) $x = 1$ is an ordinary point (b) $x = 1$ is a regular singular point
(c) $x = 1$ is an irregular singular point (d) $x = 0$ is an ordinary point

(vii) Which of the following is an odd function of t

(a) $\sin(t^2)$ (b) $t^2 - t$ (c) $t^3 + t^2$ (d) $|t|$

(viii) P.I. of $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = \sin x$ is

(a) $\sin x$ (b) $-\sin x$ (c) $\cos x$ (d) $-\cos x$

(ix) Complete solution of $pq + p + q = 0$ is

(a) $ax - \frac{a}{1+a}y + c$ (b) $ax - \frac{a}{1+a}x + c$

(c) $ax - \frac{a^2}{1+a}y + c$ (d) $ay - \frac{a}{1+a}y + c$

(x) The value of $\lim_{z\to i}\frac{iz+1}{z-i}$ is

(a) 1 (b) -1 (c) i (d) $-i$.

Group – B

2. (a) Apply residue theorem to evaluate $\int_0^{2\pi}\frac{\cos 3\theta}{5-4\cos\theta}d\theta$.

(b) Evaluate $\oint_C \frac{z^2-z+1}{z-1}dz$, where C is the circle (i) $|z| = 2$ and (ii) $|z| = \frac{1}{2}$.

6 + 6 = 12

3. (a) Find the imaginary part $v(x,y)$ of the analytic function $f(z) = u(x,y) + iv(x,y)$ Where $u(x,y) = e^{2x}(x\cos 2y - y\sin 2y)$

(b) For the function $f(z) = (z + \bar{z})(z - \bar{z}).\bar{z}$, determine the points where the Cauchy-Riemann equations are satisfied.

(c) Check if the function $f(z) = e^z$ satisfies Cauchy-Riemann equation.

6 + 4 + 2 = 12

Group – C

4. (a) Obtain the Fourier Series for the function $f(x)$ defined by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, 0 \le x \le \pi \end{cases}$$ and hence evaluate $1 + \frac{1}{3^2} + \frac{1}{5^2}.........\infty$.

(b) Using Parseval's identity for the following function

$$f(x) = \begin{cases} -x, -2 \le x \le 0 \\ x, 0 \le x \le 2 \end{cases}$$

prove that $\sum\frac{1}{n^4} = \frac{\pi^4}{96}$, $n = 1,3,5.....$

6 + 6 = 12

5. (a) Evaluate $F^{-1}[e^{-a|s|}], a > 0$.

(b) Find the Fourier sine and cosine transform of $f(x)$ defined by $f(x) = xe^{-ax}, a > 0$.

6 + 6 = 12

Group – D

6. (a) Find the series solution of the differential equation

$$(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - xy = 0$$

about the point $x = 0$.

(b) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

7 + 5 = 12

7. (a) Solve the equation $xy'' + 3y' + (1+x)y = 1 + x^2$, $y(0) = 1, y(4) = 0$ at $x = 1,2,3$.

(b) Express $f(x) = 4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials.

7 + 5 = 12

Group – E

8. (a) Derive the partial differential equation (by eliminating the arbitrary constants) from the relation

$$f(x^2 + y^2, z - xy) = 0$$

(b) Solve $x^2(y - z)p + y^2(z - x)q = z^2(x - y)$.

6 + 6 = 12

9. (a) Find the general solution of the following partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 7\frac{\partial^2 z}{\partial x \partial y} + 12\frac{\partial^2 z}{\partial y^2} = e^{x-y}$$

(b) Solve the heat equation given by

$$\frac{\partial u}{\partial t} = \frac{1}{h^2}\frac{\partial^2 u}{\partial x^2}$$

Given that $u(0,t) = 0, u(l,t) = 0, u(x,0) = \sin\frac{\pi x}{l}$.

By the method of separation of variables.

5 + 7 = 12

Note:

1. Students having backlog in MATH2001 (old syllabus) and if not joined in any Google classroom for this paper code yet, are advised to follow both Step-I and Step-II as mentioned below in order to submit the answer-scripts properly.

2. Students who have already joined any Google Classroom for MATH2001 (old syllabus) can directly go to Step-II as mentioned below.

Department & Section	Steps	Link
ME (Backlog)	Step-I : Join Google Classroom using institutional email account	https://classroom.google.com/c/Mzc1OTA0MDI2ODIx?cjc=cbfh6oj
	Step-II: Submit the answer script.	https://classroom.google.com/c/Mzc1OTA0MDI2ODIx/a/Mzc1OTA1OTYxNTIz/details