GRAPH THEORY AND ALGEBRAIC STRUCTURES (MATH 2203)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A group contains 12 elements then the possible number of elements in a subgroup is
(a) 3 (b) 5 (c) 7 (d) 11.

(ii) Which of the following set is closed under numerical multiplication
(a) {1,-1,0,2} (b) {1,i} (c) $\{1, \omega, \omega^2\}$ (d) $\{\omega, 1\}$.

(iii) The number of elements in the symmetric group S_5 is
(a) 5 (b) 20 (c) 120 (d) 5^5

(iv) Let G be a cyclic group containing 10 elements with generator x. Then x^{10} is
(a) x (b) x^{-1} (c) identity (d) x^9.

(v) Which of the following group is not cyclic?
(a) (Z, +) (b) $(\{2n: n \in Z\}, +)$ (c) (Q,+) (d) $(Z_n, +)$
(Z and Q denote the set of integers and rationals respectively.)

(vi) Travelling Salesman problem is associated with
(a) Euler's Circuit (b) Hamiltonian Circuit
(c) Regular Graph (d) Isomorphic Graph.

(vii) A square is a planar graph and has regions
(a) 0 (b) 1 (c) 2 (d) 3.

(viii) The chromatic number of K_3 is
(a) 2 (b) 3 (c) 4 (5) 5.

(ix) The number of binary operations required for a set to be a ring is
(a) 1 (b) 2 (c) 3 (d) 0.

(x) If the set of residue class modulo n is an integral domain then n is
(a) an odd integer (b) an even integer
(c) 1 (d) a prime integer.

Group – B

2. (a) Let G be non-empty graph. Prove that if G is bipartite, $\chi(G)=2$.

(b) Prove that for a complete graph K_n with n vertices, $\chi(K_n)=n, \forall n \geq 1$.

(c) Define dual of a graph. Draw the dual graph of K_4.

3 + 3 + (2 + 4) = 12

3. (a) State and prove Euler's formula.

(b) What is the maximum number of edges in a planar graph with 10 vertices?

(c) If a graph has 250 faces and 300 edges, can it be planar?

6 + 3 + 3 = 12

Group – C

4. (a) Give an example of binary operation with suitable explanation for each of the following cases:
(i) non-associative
(ii) non-commutative.

(b) Write down the multiplicative Cayley Table for U(8).

(c) Let G be group. If $a, b \in G$, then prove that $(ab)^{-1}=b^{-1}a^{-1}$.

(d) Give two examples of finite groups of order at least 4.

(2 + 2) + 3 + 3 + (1 + 1) = 12

5. (a) Let $A=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$ be two permutations. Show that $AB \neq BA$.

(b) Find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{pmatrix}$.

(c) Show that the set G of all ordered pairs (a,b) with $a \neq 0$, of real numbers a, b forms a group with operation '$\circ$' defined by, $(a,b) \circ (c,d)=(ac, bc+d)$

4 + 2 + 6 = 12

Group – D

6. (a) Prove that every subgroup of a cyclic group is cyclic.

(b) Let G be a group. If $a,b \in G$ such that $a^4=e$, the identity element of G and $ab=ba^2$, prove that $a=e$.

6 + 6 = 12

7. (a) Let $G=\langle a \rangle$ be a cyclic group of order n. Then for any integer r, $1 \leq r < n$, a^r is a generator of G iff $gcd(r,n) = 1$.

(b) Prove that any two right or left cosets of a subgroup are either disjoint or identical.

6 + 6 = 12

Group – E

8. (a) An element a of a ring R is idempotent if $a^2=a$.
   (i) Show that the set of all idempotent elements of a commutative ring is closed under multiplication.
   (ii) Find all idempotent elements in the ring $Z_6 \times Z_2$.

   (b) Let R be a ring and let a be a fixed element of R. Let $I_a = \{x \in R : ax = 0\}$. Show that I_a is a subring of R.

(3 + 4) + 5 = 12

9. (a) Define characteristic of a ring R. Let R be a commutative ring with unity of characteristic 4. Compute and simplify $(a+b)^4$ for a,b in R.

   (b) Define divisors of zero in a ring R.

   (c) Solve the equation $x^2+2x+4=0$ in Z_6.

(2 + 5) + 2 + 3 = 12

Note:

1. **Students having backlog in MATH2203 (IT) (old syllabus) and if not joined in any Google classroom for this paper code yet, are advised to follow both Step-I and Step-II as mentioned below in order to submit the answer-scripts properly.**

2. **Students who have already joined any Google Classroom for MATH2203 (IT) (old syllabus) can directly go to Step-II as mentioned below.**

Department & Section	Steps	Link
IT (Backlog)	Step-I : Join Google Classroom using institutional email account	https://classroom.google.com/c/Mzc1OTE3NDg3ODI0?cjc=eebzmyp
	Step-II: Submit the answer script.	https://classroom.google.com/c/Mzc1OTE3NDg3ODI0/a/Mzc1OTE4MjU1NDE5/details