ADVANCED COMPUTATIONAL MATHEMATICS AND GRAPH THEORY (MATH 4282)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The fifth Bernoulli number $B_5 =$

(a) $-\frac{1}{30}$ (b) $-\frac{1}{2}$ (c) 0 (d) $\frac{1}{5}$

(ii) $(1! + 2! + 3! + 4! + \cdots + 400!) \bmod 6 =$

(a) 4 (b) 3 (c) 5 (d) 1

(iii) $\varphi(31) + \varphi(11)$

(a) 40 (b) 30 (c) 10 (d) 42

(iv) $\begin{bmatrix} 6 \\ 1 \end{bmatrix} =$

(a) 720 (b) 6 (c) 24 (d) 120

(v) The generating function of the sequence $\{1, -3, 9, -27, 81, \dots, (-1)^n 3^n, \dots\}$ is

(a) $\frac{1}{1-3x}$ (b) $\frac{1}{(1+3x)^2}$ (c) $\frac{1}{1+3x}$ (d) $\frac{1}{1+x^3}$

(vi) Which one of the following is a Fibonacci number?

(a) 89 (b) 235 (c) 146 (d) 36

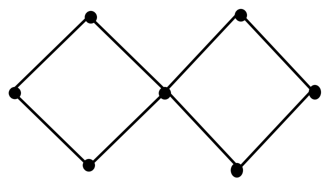
(vii) The Eulerian number $\left\langle {3 \atop 2} \right\rangle$ is

(a) 4 (b) 3 (c) 2 (d) 1

(viii) $\Delta(x^{\underline{3}}) =$

(a) $2x^2 - 3x$ (b) $3x^2 + 2x$ (c) $3x^2 - 3x$ (d) $3x^2 + 3x$

(ix) The chromatic number of the following graph is



(a) 2 (b) 10
(c) 5 (d) 3

(x) If $\omega(G)$ denotes the number of components in a graph G and an edge e be a bridge in G then
(a) $\omega(G-e)=\omega(G)$ (b) $\omega(G-e)=\omega(G)+1$
(c) $\omega(G-e)<\omega(G)+1$ (d) $\omega(G-e)=1$

Group – B

2\. (a) Consider the 'Tower of Hanoi' problem. Let T_n denote the minimum number of moves that will transfer n disks from one peg to another if only one disk is moved at a time and a larger one is never moved onto a smaller one. Prove that $T_n \leq 2T_{n-1}+1$, for $n>0$. Explain why the formula uses '$\leq$' instead of '$=$'.

(b) (i) Prove that $\Delta(x^{\underline{m}}) = mx^{\underline{m-1}}$.
(ii) Let $\nabla f(x) = f(x) - f(x-1)$. What is $\nabla\left(x^{\overline{m}}\right)$? Prove your answer.

6 + (3 + 3) = 12

3\. (a) In the Josephus problem involving n people numbered 1 to n around a circle, let $J(n)$ denote the survivor's number. State the recurrence obeyed by $J(n)$. Calculate $J(n)$ for $n = 1,2,3,4,\dots,15,16$. Prove that $J\,(2^m+l) = 2l+1$, where l is even.

(b) Find a necessary and sufficient condition that $\lfloor nx\rfloor = n\lfloor x\rfloor$, when n is a positive integer.(Your condition should involve $\{x\}$.)

6 + 6 = 12

Group – C

4\. (a) Prove the following recurrence formula for $\left\{ n \atop k \right\}$, the Stirling numbers of the second kind :
$\left\{ n \atop k \right\} = k\left\{ n-1 \atop k \right\} + \left\{ n-1 \atop k-1 \right\}$, integer $n>0$.

(b) Let B_n denote the n-th Bernoulli number. Prove that $B_{2m+1}=0$ for all integers $m \geq 1$.

6 + 6 = 12

5\. (a) State the binomial theorem. Use it to prove that the number of subsets of a set having n elements is 2^n.

(b) State the definition of the Eulerian numbers $\left\langle n \atop k \right\rangle$. Calculate $\left\langle 4 \atop 2 \right\rangle$, $\left\langle 4 \atop 1 \right\rangle$, $\left\langle 4 \atop 3 \right\rangle$ by considering the set $\{1,2,3,4\}$. Show your work in detail.

6 + 6 = 12

Group – D

6\. (a) Find the greatest common divisor of 188 and 119 by using the Euclidean algorithm. Express it as $188x + 119y$ where x and y are integers. Show your work in detail.

(b) Find $(2^{402} + 3^{501} + 5^{802})$ mod 17. Show your calculations in detail and state any theorem that use.

6 + 6 = 12

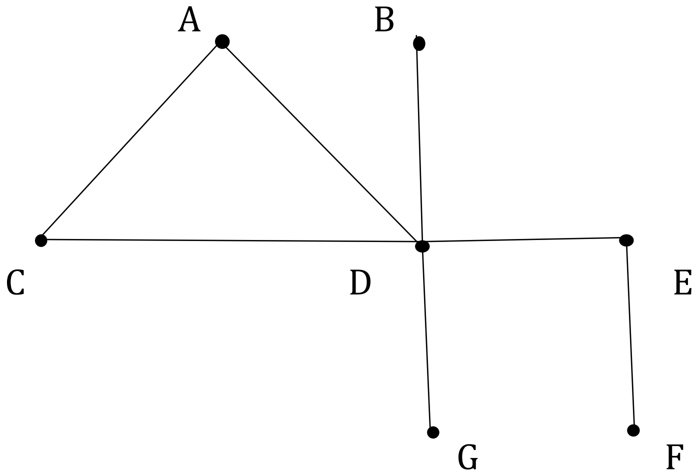
7. (a) Prove that (i) $\lfloor x \rfloor = n \Leftrightarrow x - 1 < n \leq x$ and (ii) $x < n \Leftrightarrow x - 1 < n \leq x$, where n is an integer and x is a real number.

(b) Let $\varphi(n)$ denote the number of integers amongst $\{0,1,2,\ldots,m-1\}$ that are relatively prime to n. Let p be a prime and k be a positive integer.
(i) Prove that $\varphi(p^k) = p^k - p^{k-1}$. (ii) Find $\varphi(72)$.
Show your calculation in detail.

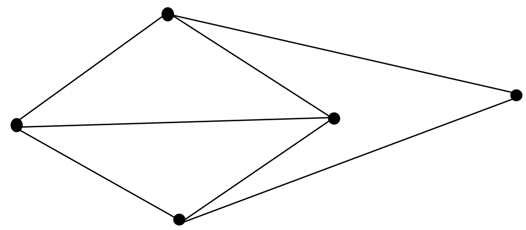
6 + 6 = 12

Group – E

8. (a) Find four independent sets of vertices and independence number of the following graph:

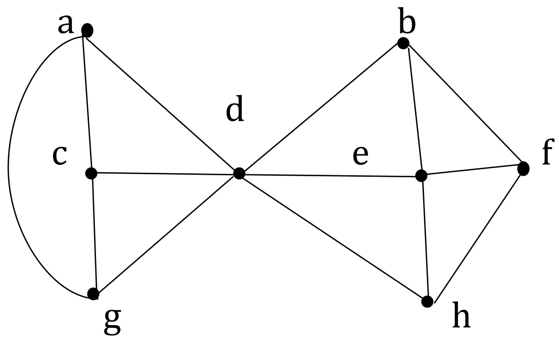


(b) Apply Decomposition theorem to find the chromatic polynomial of the following graph:



4 + 8 = 12

9. (a) Define edge connectivity and vertex connectivity of a graph. Find the edge connectivity and vertex connectivity of the graph given below:



(b) Applicants a_1, a_2, a_3 and a_4 apply for five posts $p_1, p_2, p_3, p_4$and p_5. The applicants are done as follows : $a_1 \to \{p_1, p_2\}, a_2 \to \{p_1, p_3, p_5\}, a_3 \to \{p_1, p_2, p_3, p_5\}$ and $a_4 \to \{p_3, p_4\}$. Using Hall's marriage theorem find

(i) Whether there is a perfect matching of the set of applicants into the set of objects?

(ii) If yes, find a matching where every applicant can be offered a single post.

6 + 6 = 12

Department & Section	Submission Link
AEIE & CSE	https://classroom.google.com/c/Mjk4ODM5NTg1NTE3/a/MzQwNjI3NTc4Njk4/details