FUNDAMENTALS OF DIGITAL SIGNAL PROCESSING (AEIE 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) For a system, $y(n) = nx(n)$, the inverse system will be
(a) $y\left(\frac{1}{n}\right)$ (b) $\frac{1}{n}y(n)$
(c) $ny(n)$ (d) $n^{-1}y(n)$

(ii) Which of the following system is causal?
(a) $h(n) = n\left(\frac{1}{2}\right)^n u(n+1)$ (b) $y(n) = x^2(n) - x(n+1)$
(c) $y(n) = x(-n) + x(2n-1)$ (d) $h(n) = n\left(\frac{1}{2}\right)^n u(n)$

(iii) If all the poles of the system function $H(z)$ have magnitude smaller than 1, then the system will be
(a) stable (b) unstable
(c) BIBO stable (d) a and c

(iv) The Z-transform of $a^{-n}u(-n-1)$ is
(a) $\frac{-z}{z-\frac{1}{a}}$ (b) $\frac{z}{z-\frac{1}{a}}$
(c) $\frac{z}{z-a}$ (d) $\frac{-z}{z-a}$

(v) The inverse z-transform of $\frac{3}{z-4}$, $|z| > 4$ is
(a) $3(4)^n u(n-1)$ (b) $3(4)^{n-1}u(n)$
(c) $3(4)^{n-1}u(n+1)$ (d) $3(4)^{n-1}u(n-1)$

(vi) If $DFT\{x(n)\} = X(k)$, then $DFT\{x(n+m)_n\}$ will be
(a) $X(k)e^{\frac{-j2\pi km}{N}}$ (b) $X(k)e^{\frac{-j2\pi k}{mN}}$
(c) $X(k)e^{\frac{j2\pi km}{N}}$ (d) $X(k)e^{\frac{j2\pi k}{mN}}$

(vii) The normalized transfer function of 3rd order low pass Butterworth filter is

(a) $\frac{1}{s^3+1.4141 s_n^2+s_n+1}$ (b) $\frac{1}{(s_n+1)(s_n^2+s_n+1)}$

(c) $\frac{1}{s^2(s_n+1)}$ (d) $\frac{1}{s_n^3+s_n^2+s_n+1}$

(viii) The poles of Butterworth transfer function symmetrically lies on a circle in s-plane with angular spacing

(a) $\frac{\pi}{N}$ (b) $\frac{\pi}{2N}$

(c) $\frac{2\pi}{N}$ (d) $\frac{\pi}{N^2}$

(ix) In impulse invariant transformation the analog system with transfer function, $H(s) = \frac{0.3}{s+0.7}$ is transformed to a digital system with transfer function

(a) $\frac{-0.3}{1-e^{-0.7T}z^{-1}}$ (b) $\frac{0.3}{1-e^{-0.7T}z^{-1}}$

(c) $\frac{0.7}{1-e^{-0.3T}z^{-1}}$ (d) $\frac{-0.7}{1-e^{-0.3T}z^{-1}}$

(x) The frequency response of a digital filter is periodic in the range

(a) $0 < \omega < 2\pi$ (b) $-\pi < \omega < \pi$

(c) $0 < \omega < \pi$ (d) $0 < \omega < 2\pi$ or $-\pi < \omega < \pi$

Group – B

2. (a) Determine if the signal $x(n) = e^{\frac{j7\pi n}{4}}$ is periodic or not. If periodic find the fundamental period.

(b) Determine the even and odd part of the signal $x(n) = 2e^{\frac{j\pi n}{3}}$.

(c) What are energy and power signals?

(d) Determine whether the signal $x(n) = e^{j\left(\frac{\pi}{2}n+\frac{\pi}{4}\right)}$ is energy signal, power signal or none of them.

3 + 3 + 2 + 4 = 12

3. (a) Determine the linear convolution of following sequences using Overlap add method:
$x(n) = \{3, -1, 0, 1, 3, 2, 0, 1, 2, 1\}$; $h(n) = \{1, 1, 1\}$.

(b) Perform cross correlation of the sequences, $x(n) = \{1, 1, 2, 2\}$ and $(n) = \{1, 0.5, 1\}$.

8 + 4 = 12

Group – C

4. (a) Determine the z-transform and sketch the ROC of the following signal

$$x(n) = \begin{cases} \left(\frac{1}{3}\right)^n, & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n}, & n < 0 \end{cases}$$

(b) Find the input $x(n)$ of the system, if the impulse response $h(n)$ and the output $y(n)$ are as shown below:
$h(n) = \{1, 2, 3, 2\}; y(n) = \{1, 3, 7, 10, 10, 7, 2\}$.

(c) What is the relationship between Z-transform and Fourier transform?

5 + 5 + 2 = 12

5. (a) Find the one-sided z-transform of the discrete time signal generated by mathematically sampling the following continuous time signal $x(t) = \sin \Omega_0 t$.

(b) Determine the inverse z-transform of the function, $X(z) = \frac{3+2z^{-1}+z^{-2}}{1-3z^{-1}+2z^{-2}}$ by the following two methods and prove that the inverse z-transform is unique.
i. Residue Method
ii. Partial Fraction Expansion Method.

4 + (4 + 4) = 12

Group – D

6. (a) Compute 8-point DFT of the following sequence using DIT algorithm:

$$x(n) = \begin{cases} 2, \text{ for } n = 0, 2, 5, 7 \\ 1, \text{ for } n = 1,3, 4, 6 \\ 0, \text{ elsewhere} \end{cases}$$

(b) Compute the DFT of a sequence $x(n) = \delta(n)$.

10 + 2 = 12

7. (a) Given the sequences $x_1(n) = \{1, 2, 3, 4\}$; $x_2(n) = \{1, 1, 2, 2\}$. Find $x_3(n)$ such that $X_3(k) = X_1(k)\, X_2(k)$ where $X(k) = DFT\{x(n)\}$.

(b) Determine the IDFT of the sequence $X(k) = \{3, -j, 1, j\}$.

(c) Show that with $x(n)$ as an N-point sequence and $X(k)$ as its N-point DFT,

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2$$

4 + 4 + 4 = 12

Group – E

8. Design a Butterworth digital IIR low pass filter using bilinear transformation by taking T=0.1 second to satisfy the following specifications:
$0.6 \leq |H(j\Omega)| \leq 1$ for $0 \leq \Omega \leq 0.35\pi$
$|H(j\Omega)| \leq 0.1$ for $0.7\pi \leq \Omega \leq \pi$
Draw direct form-I structure of the filter.

8 + 4 = 12

9. (a) Realize the following system with minimum number of multipliers:

$$H(z) = (1 + z^{-1}) \left(1 + \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} + z^{-3}\right)$$

(b) Distinguish between FIR and IIR filters.

(c) What condition on the FIR sequence $h(n)$ are to be imposed in order that this filter can be called a linear phase filter?

7+ 3 + 2 = 12

Department & Section	Submission Link
AEIE	https://classroom.google.com/u/1/w/MzY0NzY4NjM2Njk3/tc/MzY0NzY4OTk5MTgw