PHYSICS - II
(PHYS 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If the generalised coordinate q_j is cyclic, then the corresponding generalised momentum p_j is

(a) $\frac{\delta L}{\delta q_j}$ (b) $\frac{d}{dt}(\frac{\delta L}{\delta q_j})$ (c) $\frac{\delta L}{\delta \dot{q}_j}$ (d) $\frac{d}{dt}(\frac{\delta L}{\delta \dot{q}_j})$

(ii) The number of possible arrangements of two Fermions in 3 cells are

(a) 6 (b) 5 (c) 4 (d) 3

(iii) The lagrangian of a system is a function of

(a) q_j, p_j, t (b) $q_j, \dot{q}_j, t$ (c) $\dot{p}_j, q_j, t$ (d) $p_j, \dot{q}_j, t$

(iv) The density of states for fermions is proportional to

(a) E (b) E^2 (c) $E^{\frac{1}{2}}$ (d) $\frac{1}{E}$

(v) The solution of Schrodinger equation for an electron moving in a periodic potential is of the form

(a) $\psi_k(\mathbf{r}) = e^{i\mathbf{k.r}}$ (b) $\psi_k(\mathbf{r}) = u_k(\mathbf{r})\, e^{i\mathbf{k.r}}$

(c) $\psi_k(\mathbf{r}) = u_k(\mathbf{r})$ (d) $\psi_k(\mathbf{r}) = u_k(\mathbf{r}) + A\, e^{i\mathbf{k.r}}$.

(vi) Which of the following function is eigen function of the operator d^2/dx^2

(a) $\psi = c \ln x$ (b) $\psi = cx^2$ (c) $\psi = c\, e^{-mx}$ (d) $\psi = c/x$

(vii) The degrees of freedom for a system of N particles with K constraint relations is given by

(a) N-K (b) 3N- K (c) N- 3K (d) 3K - N.

(viii) The average velocity of an electron at T = 0K in a metal in terms of velocity v_f at the Fermi level is

(a) $\frac{2}{3}v_f$ (b) $\frac{1}{3}v_f$ (c) $\frac{3}{2}v_f$ (d) $\frac{3}{4}v_f$

(ix) The density of charge carrier in a pure semiconductor is proportional to

(a) $e^{\frac{-E_g}{KT}}$ (b) $e^{\frac{-E_g}{2KT}}$ (c) $e^{\frac{-2E_g}{KT}}$ (d) $e^{\frac{-E_g}{T}}$

(x) For a particle trapped in a box of length L, the average value of momentum <P> is

(a) 0 (b) 1 (c) ½ (d) L.

Group – B

2. (a) A two particle system in three Cartesian dimensions admits four holonomic constraints.
   (i) How many Lagrange equations are possible for the system?
   (ii) What is the dimension of its configuration space?

   (b) The kinetic and potential energy of a system in terms of generalized coordinates$\{q_{1,}q_2\}$ are given by:
   $T = \alpha\dot{q}_1^2\dot{q}_2 + \beta q_1\dot{q}_2^2$ & $V = K\cos(q_1)$; α, β *and K are constant*.
   (i) Construct the lagrangian. (ii) Find the components of generalized momentum. (iii) Identify the cyclic coordinate if any. (iv) Construct the Lagrange equation of motion

   (c) Find out Hamilton's equations of mass for a system comprising of masses m_1 and m_2 connector by a mass less spring of length L through a frictionless pulley such that $m_1 > m_2$.

(1 + 1) + (2 + 2 + 1 + 2) + 3 = 12

3. (a) Find the eigenfunction of the momentum operator
   $\widehat{P_x}$= - i$\hbar\frac{d}{dx}$, corresponding to the eigenvalue p.

   (b) Evaluate the commutator $[\hat{x}, \hat{L}_y]$.

   (c) Find the expectation value of 'x' of a particle confined in one-dimensional box of length 'a' if the wave function for the ground state is
   $\varphi_n = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$

   (d) Normalize the following wave function, $\Psi(x) = A\sin\frac{n\pi x}{L}$ where, $0 \leq x \leq L$.

   (e) Show that if a coordinate is cyclic in Lagrangian it will be cyclic in Hamiltonian also.

2 + 3 + 3 + 2 + 2 = 12

Group – C

4. (a) Write down the Fermi-Dirac distribution function and define Fermi level in metal at absolute zero and at finite temperature.

(b) Four particles are distributed into three energy levels having energies 0, E, 3E so that the total energy is 4E. If the levels are degenerate with degeneracy 2, 2, 3 respectively, find out the macrostates and the corresponding microstates for M-B particles, F-D particles and B-E particles.

(c) Apply BE statistics to photon gas and deduce Planck's law of spectral energy density of black body radiation.

(1 + 2) + (2 + 2 + 2) + 3 = 12

5. (a) Find out the expression of average speed [v_{avg}] of ideal gas molecules. Given that $N(E)dE = \left[\frac{2\pi N}{(\pi KT)^{\frac{3}{2}}}\right] e^{-\frac{E}{KT}} E^{\frac{1}{2}} \; dE$, where symbols have their meaning.

(b) Express the Fermi energy value in a metal in terms of free electron density at T=0K.

(c) Find the Fermi energy in copper on the assumption that each copper atom contributes one free electron to electron gas. [Given that: density of copper 8.94 gm/cm^3, Atomic mass of copper 63.5].

(d) Using Fermi-Dirac distribution, Calculate the concentration of holes in the valance band of an intrinsic semiconductor.

3 + 2 + 3 + 4 = 12

Group – D

6. (a) Derive the relation between the electric susceptibility of a dielectric and its atomic polarizability.

(b) Calculate the induced dipole moment per unit volume of He gas if it is placed in an electric field of 6000 V/cm. The atomic polarizability of He is $0.18 \times 10^{-40}\, Fm^2$ and gas density is 2.6×10^{25} atoms per m^3.

(c) Find the expression of induced charge in a dielectric material when it is kept within a parallel plate capacitor?

(d) Write down the relation between electric displacement vector and polarization vector.

4 + 3 + 3 + 2 = 12

7. (a) The magnetic susceptibility of Cu is - 0.5×10^{-5}. Calculate the magnetic moment per unit volume in Cu when subjected to a field whose magnitude inside copper is 10^4 A/m. [Given: $\mu_0 = 4\pi \times 10^{-7}$ H].

(b) Draw $\frac{1}{\chi}$ versus T [Where, χ is the susceptibility of the material] for paramagnetic, ferromagnetic and anti-ferromagnetic materials on the same graph.

(c) What is meant by ferromagnetic materials? Give some examples. What are their characteristics?

(d) What is retentivity and coercivity, explain with B-H curve.

2 + 3 + (1 + 1 + 2) + 3 = 12

Group – E

8. (a) The energy-wave vector dispersion relation for a one dimensional crystal of lattice constant 'a' is given by $E(\kappa) = E_0 + 3\alpha\kappa^2 - 5\beta\kappa^4$, where E_0, α, β are positive constants. Find the expression for the velocity of the electron as a function of κ. For what value of κ the velocity is maximum?

(b) Calculate the number density of electrons in a material for which London penetration depth is $\lambda = 0.5 \times 10^{-8} m$.(take $\mu_0 = 4\pi \times 10^{-7}$ in SI unit).

(c) Distinguish between types I and type II superconductors. Name some materials belonging to these two types of superconductors.

(3 + 2) + 3 + (2 + 2) = 12

9. (a) Show that when an electron moves through a crystal, the effective mass of the electron can be expressed as

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

Where the symbols have their usual meaning.

(b) The energy-wave vector dispersion relation for a one dimensional crystal of lattice constant 'a' is given by $E(\kappa) = E_0 - \alpha - 2\beta \cos \kappa a$, where E_0, α, β are constants. Obtain the effective mass of the electron at the bottom and the top of the band.

(c) State and explain Bloch's theorem. What is the outcome of the theory?

(d) Establish London equation of superconductivity in terms of magnetic field induction.

3 + 2 + (3 + 1) + 3 = 12

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