MATHEMATICAL METHODS IN CHEMICAL ENGINEERING
(CHEN 3204)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Equations of the form Ax = b have infinitely many solutions if
(a) Rank of A and augmented matrix A|b are same and equal to n
(b) Rank of A and A|b are same and less than n
(c) Rank of A is less than that of A|b
(d) Rank of A|B is equal to n and rank of A is less than n

(ii) If the sequence of polynomials $P_j(\lambda)$ form a Sturm sequence, V(a) be the number of sign changes in the sequence for λ=a, V(b) the number of sign changes for λ=b, the number of real zeros of $P_n(\lambda)=0$ in the range $a<\lambda<b$
(a) V(b) (b) V(a)
(c) V(b)-V(a) (d) Independent of sign changes

(iii) The differential equation representing temperature distribution in a triangular fin given by $\frac{d^2T}{dx^2}+\frac{1}{x}\frac{dT}{dx}-\frac{2Lh}{ktx}T=0$ is
(a) 2nd order 2nd degree nonlinear differential equation
(b) 2nd order 1st degree linear differential equation
(c) 2nd order 1st degree nonlinear differential equation
(d) 2nd order 2nd degree linear differential equation

(iv) A series reaction $A\rightarrow B\rightarrow C$ occurs in an isothermal batch reactor. The reactions are first order. Model equations representing the system are
(a) linear differential equations (initial value problem)
(b) linear algebraic equations
(c) linear differential equations (boundary value problem)
(d) partial differential equations

(v) In batch chemical reactors when materials undergo chemical reactions by two or more simultaneous second order paths, the equations arising are
(a) Homogenous differential equations of first order
(b) Linear Algebraic equations
(c) Homogenous differential equations of second order
(d) Nonlinear algebraic equations

(vi) In a heat conduction problem the temperature varies with position and time according to T = T(x, y, t). The correct representation of the problem is
(a) $\frac{\partial T}{\partial t} = \alpha \frac{\partial 2T}{\partial x2}$ (b) $\frac{\partial T}{\partial t} = \alpha \frac{\partial 2T}{\partial y2}$
(c) $\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial 2T}{\partial x2} + \frac{\partial 2T}{\partial y2}\right)$ (d) $\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial 2T}{\partial x2} + \frac{\partial 2T}{\partial y2} + \frac{\partial 2T}{\partial z2}\right)$

(vii) Navier-Stokes equation is an example of:
(a) linear PDE (b) linear ODE
(c) non-linear PDE (d) non-linear ODE

(viii) At the point where boundary layer separates
(a) $\tau_w = 0$ (b) $\tau_w = \infty$
(c) Normal stress = ∞ (d) The flow is irrotational

(ix) The correct order of boundary layer thicknesses from smallest to largest is
(a) $\delta < \delta_d < \delta_m$ (b) $\delta_d < \delta < \delta_m$
(c) $\delta_m < \delta_d < \delta$ (d) $\delta < \delta_m < \delta_d$

(x) For large values of Reynolds number the disturbance thickness (**δ**) of a boundary layer approaches
(a) 0 (b) ∞
(c) the length of the plate **(l)** (d) 0.664 **l**

Group – B

2. (a) A distillation apparatus consists of a boiler, a condenser and a constant level device fed with condenser cooling water. The steam is condensed in the condenser and collected in the receiver. Some of the latent heat of evaporation is returned to the boiler by preheating the feed. If the condenser feed rate is F kg/s, feed inlet temperature is T_0 °C, exit water temperature is T °C, excess water overflow rate is W kg/s and distillation rate is G kg/s, derive the expression for rate of collection of the distillate (D) and show the variation with feedrate with the help of a diagram.

(b) A tank contains 2 m^3 water. A stream of brine containing 20 kg/m^3 salt is fed to the tank at 0.02 m^3/s. Liquid flows from the tank at a rate 0.01 m^3/s. If the tank is well agitated, what is the salt concentration in the tank when the tank contains 4 m^3 brine?

(c) Prove that eigenvectors of a matrix and its transpose form an orthogonal set.

6 + 3 + 3 = 12

3. (a) Perform a transient stage analysis of a N-staged absorption column and obtain the eigenvalues of the system through recursive technique. Generate the eigenvectors and obtain the solution of the system.

   (b) The reversible reaction $A \Leftrightarrow B$ occurs isothermally in a batch reactor. The forward and reverse reactions are both first order with rate constants 1 s^{-1} and 2 s^{-1} respectively. The initial concentrations of x_1, x_2 are 2 gmol/cc and 3 gmol/cc. Determine the equilibrium concentration of A and B in the reactor.

8 + 4 = 12

Group – C

4. (a) Derive the temperature distribution in a transverse fin when the fluid surrounding the fin is hotter than the fin itself. Derive the expression of efficiency for such fins

   (b) Solve the following equation in series

$$9x(1-x)\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 4y = 0.$$

(5 + 3) + 4 = 12

5. (a) Two thin wall metal pipes of 2.5 cm external diameter and joined by flanges 1.25 cm thick and 10 cm diameter are carrying steam at 120° C. If the conductivity of the flange metal is 400 W/m ° C and the exposed surfaces of the flanges lose heat to the surroundings at T_1=20o C with a heat transfer coefficient 12 W/m^2 ° C, find the rate of heat loss from the pipe and the proportion which leaves the rim of the flange.

   (b) Determine the steady state temperature distribution in an infinitely long solid cylinder of radius R in which heat is generated per unit volume at the rate (a+bT) where a and b are positive constants and T the temperature. The external surface is maintained at constant temperature T_0.

7 + 5 =12

Group – D

6. (a) Frame a boundary value problem involving unsteady state heat conduction. State the appropriate conditions also.

   (b) Explain the concept of orthogonal functions giving a suitable example.

8 + 4 = 12

7. (a) Find out the concentration profile in a slab in the y direction under unsteady state if the initial concentration throughout the slab is uniform at C_0 and one face at y=0 suddenly attains a concentration C_1. Mention the appropriate initial and boundary conditions. Given $L^{-1}\left(\frac{1}{s}\, e^{-k\sqrt{s}}\right) = erfc\left(\frac{k}{2\sqrt{t}}\right)$.

(b) Give an example of the function-specified type of boundary condition applied in a momentum transfer problem

8 + 4 = 12

Group – E

8. What is the significance of Damkohler number in connection to diffusion with chemical reaction? Explain how boundary layer concept can be applied to describe the situation when diffusion takes place along with chemical reaction for an isothermal laminar flow along a flat plate.

(2 + 10) = 12

9. (a) Determine an expression for displacement thickness of a boundary layer. Explain physically the relationship between displacement thickness and disturbance thickness.

(b) How can the Prandtl boundary layer equations be modified to form an integral momentum balance equation?

(5 + 3) + 4 = 12

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CHE	https://classroom.google.com/c/MzY4NzI2NTY3ODM5/a/MzY4NzI2NTY3ODQ4/details