COMPUTATIONAL FLUID DYNAMICS
(CHEN 3231)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) In fluid dynamics, the continuum assumption holds good if ______________
(a) Knudsen number lies between 0.1 and 10
(b) Knudsen number is less than 0.01
(c) Knudsen number is greater than 10
(d) Knudsen number lies between 0.01 and 0.1

(ii) In a semi-infinite domain, the boundary condition for advection-diffusion-reaction-source equation at the exit is ______________
(a) Neumann (b) Dirichlet
(c) Periodic (d) Either (a) or (b)

(iii) For an incompressible fluid, the steady state continuity equation is ____________
(a) $\frac{\partial \rho}{\partial t} + div(u) = 0$ (b) div (ρu)=0
(c) div (u)=0 (d) $\frac{\partial \rho}{\partial t} + div(\rho u) = 0$

(iv) The general notation to define the coefficient for 'i+1'th node using upwind scheme is given by ______________, where D=Diffusive flux, F=Convective flux, (i+1)/2 represents midway between ith and (i+1)th node.
(a) $D_{(i+1)/2} + \max(0, -F_{(i+1)/2})$ (b) $D_{(i+1)/2} + \min(0, -F_{(i+1)/2})$
(c) $D_{(i+1)/2} - \max(0, -F_{(i+1)/2})$ (d) $D_{(i+1)/2} - \min(0, -F_{(i+1)/2})$

(v) To simulate an unsteady state heat conduction problem using explicit scheme the time step (Δt) must be ______________ for stable convergence.
(a) $\Delta t = \frac{\rho C_P}{2k}(\Delta X)^2$ (b) $\Delta t \le \frac{\rho C_P}{2k}(\Delta X)^2$
(c) $\Delta t > \frac{\rho C_P}{2k}(\Delta X)^2$ (d) $\Delta t < \frac{\rho C_P}{2k}(\Delta X)^2$

(vi) In hybrid scheme to find the value of property ϕ at face, central difference scheme is applied for

(a) Pe=2 (b) Pe>2 (c) Pe<2 (d) 2<Pe<10

(vii) In SIMPLE algorithm it is ________________

(a) assumed that the error at all the neighbourhood nodes are zero
(b) assumed that the summation of errors at all the neighbourhood nodes are zero
(c) assumed that the summation of errors at the evaluating node is zero
(d) assumed that the error in the convective term is equal to error in pressure

(viii) The CFL number generated from application of finite difference scheme on unsteady state convection-diffusion equation is given by ________________

(a) $\frac{|u|\Delta t}{\Delta x}$ (b) $\left[\frac{|u|\Delta t}{\Delta x}\right]^{-1}$ (c) $\frac{2D\Delta t}{(\Delta x)^2}$ (d) $\left[\frac{2D\Delta t}{(\Delta x)^2}\right]^{-1}$

(ix) To obtain the solution of Navier-Stokes equation for a fluid flowing inside a constant difference in pressure the meshing may be done with the ________ grid formation.

(a) staggered (b) collocated
(c) both (a) and (b) (d) none of these

(x) During mapping of the physical domain, ranging from near the surface till far away from the surface, on the computational domain y-axis transformation can be given by _____________ .

(a) $1-\frac{\ln\left(\frac{\beta+1-y/h}{\beta-1+y/h}\right)}{\ln\left(\frac{\beta+1}{\beta-1}\right)}$, when $\beta>1$

(b) $1-\frac{\ln\left(\frac{\beta+1-y/h}{\beta-1+y/h}\right)}{\ln\left(\frac{\beta+1}{\beta-1}\right)}$, when $\beta<1$

(c) $1+\frac{\ln\left(\frac{\beta+1-y/h}{\beta-1+y/h}\right)}{\ln\left(\frac{\beta+1}{\beta-1}\right)}$, when $\beta>1$

(d) $1+\frac{\ln\left(\frac{\beta+1-y/h}{\beta-1+y/h}\right)}{\ln\left(\frac{\beta+1}{\beta-1}\right)}$, when $\beta<1$

Group – B

2. (a) Distinguish between Lagrangian and Eulerian descriptions of fluid flow. Discuss how the equations of motion applicable for a fluid particle (Lagrangian description) can be modified suitably to be used for Eulerian description.

(b) Write the stress components on a fluid element of area δAx and show them with the help of a figure. Illustrate the energy fluxes associated with an infinitesimally small fluid element and explain the different terms. Write the Navier Stokes equation in x direction explaining each term in the equation. Under which condition does the equation reduce to Euler's equation?

(2 + 3) + (2 + 3 + 2) = 12

3. (a) What is the basic principle for derivation of the advection diffusion equation? Write down the one dimensional advection diffusion equation explaining each term. Sketch the concentration profiles of a tracer in a pipe with only molecular diffusion and with both advection and diffusion present and compare them.

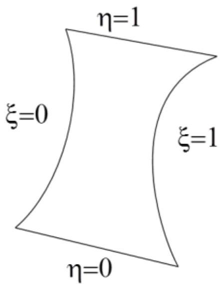
(b) Write the modified advection diffusion equation when reaction and source terms are present. Explain the significance of Peclet number in advection diffusion equation. What are the two dimensionless groups generated in the flow, dispersion and reaction equation in a packed bed reactor? Give an example of advection diffusion systems with periodic boundary conditions.

(1 + 2 + 2) + (2 + 2 + 2 + 1) = 12

Group – C

4. (a) "In case of uniform grid generation in computational domain through grid mapping algorithm from concentrated grids near the boundary of a flow domain, the derivative of the mapped variable with the actual variable must show a large value compared to the rest of the actual domain." – Justify the appropriateness of the statement with a proper schematic.

(b) Find out the coordinates of the grid points in the computational domain for the below geometry (shown in the figure) after applying the Lagrangian interpolation method. Show the grids in the computational domain with appropriate sketch.



Where ξ and η are the variables in the computational domain with $\Delta\xi = \Delta\eta = 0.25$.

2 + 10 = 12

5. (a) Find out stability condition of the backward in time-forward in space (BTFS) algorithm, when applied to solve unsteady heat conduction equation at different time interval.

(b) "For 1D unsteady state problem ADI method can be applied by choice, while with 2D unsteady state problem ADI method is the only one to simulate the problem." – Comment on the appropriateness of the statement with proper mathematical elaboration.

7 + 5 = 12

Group – D

6. (a) For a 2D convection-diffusion problem on ϕ transport, show that ϕ_P at node P of a control volume can be expressed in the form as $\phi_P = \frac{\sum a_{nb}\phi_{nb}}{a_p}$, where subscript 'nb' refers neighbouring nodes and 'a' refers to the coefficient to ϕ at a node.

(b) What is the need for staggered grid arrangement in case to simulate a steady state flow field described by Navier-Stokes equation? Elaborate with appropriate reasoning.

7 + 5 = 12

7. (a) Why the fully implicit scheme to solve an unsteady state conduction problem does not have any conditional stability criterion? Elaborate with proper mathematical reasoning.

(b) A thin plate is at 200°C uniform temperature initially. Its east face temperature is suddenly reduced to 0°C and rest of the faces are insulated. Find the distribution at t=16 s using Crank-Nicolson scheme (generate 5 nodes).
Given: Thermal conductivity (k)=10 W/mK; $\rho C_P = 10^7$ J/m^{3}K; Length of the plate = 0.02 m

3 + 9 = 12

Group – E

8. (a) "For a pressure-velocity coupled flow field, the staggered grid conformation becomes useful instead of collocated grid." – Justify the appropriateness of the statement with relevant mathematical expression.

(b) The free stream velocity of water inside two parallel flat plates kept 0.1 m distance apart is 5 m/s. The gauge reading shows that there is a pressure drop of 0.5 kgf/m^2, for the entire flow path length of 1 m. Consider the flow is laminar and the surface of the plates are very smooth. The width of the plate is equal to 0.5 m. The kinematic viscosity of water = 7.23×10^{-7} m^2/s. Calculate the pressure and velocity profile within the plate using SIMPLE algorithm (show one iteration).

3 + 9 = 12

9. (a) "Pressure correction approach is more generic compared to artificial compressibility method in solving the flow domain for incompressible steady state viscous flow" – Justify the correctness of the statement.

(b) During the flow of water inside a squared cross-section pipeline of 1 m and dimension 0.15 m × 0.15 m at 35°C, one has suddenly made the flow velocity '0' by obstructing the flow at one end of the pipe. Find out the pressure built up at the obstruction point once the flow ceases using artificial compressibility algorithm (show two iterations). The initial velocity of water 0.1 m/s. ρ_{water}=1000 kg/m^3; μ_{water}=0.001 Pa.s, Sound velocity at this temperature through water medium = 1520 m/s. Assume 3 nodes along the length and 2 nodes along the height.

3 + 9 = 12

Department & Section	Submission Link
CHE	https://classroom.google.com/c/Mjk2ODgwNzIzMzQ3/a/MzU3NDU0NjY2NzUx/details