TRANSPORT PHENOMENA
(CHEN 2202)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) Thermal diffusivity is given by ________________

(a) $\frac{k}{\rho C_p}$ (b) $\frac{k}{\rho C_v}$

(c) $\frac{k\rho}{C_p}$ (d) $\frac{k\rho}{C_v}$

(ii) Momentum flux for i=j is given by ________________

(a) $\phi_{ij} = p + \rho u_i u_j$ (b) $\phi_{ij} = p + \tau_{ij} + \rho u_i u_j$

(c) $\phi_{ij} = \tau_{ij} + \rho u_i u_j$ (d) $\phi_{ij} = \rho u_i u_j$

(iii) For irrotational flow,

(a) $\nabla \times u = 0$ (b) $\nabla . u = 0$

(c) $\nabla .(\nabla . u) = 0$ (d) $\nabla \times (\nabla \times u) = 0$

(iv) Reynold's Analogy is valid for _____________

(a) Re=0.1 (b) Re=10

(c) Re=1 (d) 0.1<Re<1

(v) For heat transfer process, Peclet number (Re×Pr) tells us about the ratio between ___________ and _____________ within the boundary layer

(a) convective energy transport, conductive energy transport

(b) convective energy transport, radiative energy transport

(c) convective energy transport, convective mass transport

(d) convective energy transport, diffusive mass transport

(vi) Thermal diffusivity is given by ____________

(a) Cpμ/k (b) k/ρCp

(c) ρCp/k (d) k/ρCv

(vii) Lewis number ($k/\rho C_p D_{AB}$) tells us about the ratio between ____________ and ___________ diffusivities
(a) thermal, momentum
(b) mass, momentum
(c) mass, thermal
(d) thermal, mass

(viii) Using Von-Karman integral method the thickness of momentum boundary layer is equal to _________________
(a) $\frac{4.64}{\sqrt{Re_x}}$ (b) $\frac{2.32}{\sqrt{Re_x}}$ (c) $\frac{1}{\sqrt{Re_x}}$ (d) $\frac{9.28}{\sqrt{Re_x}}$

(ix) Hagen-Poiseuille equation for volumetric flow rate determination of a fluid flow inside a pipe of radius R and length L is given by ________________
(a) $\frac{\pi\Delta P}{8\mu L}R^4$ (b) $\frac{\pi\Delta P}{8\mu L}R^2$ (c) $\frac{\pi\Delta P}{8\mu L}R^3$ (d) $\frac{\pi\Delta P}{8\mu L}R$

(x) Stanton number for mass transfer is expressed as:
(a) Pr/Re Sc
(b) Nu/ Re Pr
(c) Sh/Pr Sc
(d) Sh/Re Sc

Group – B

2. (a) Define a spatial vector field with a suitable example.

(b) Compute the steady state momentum flux, when the upper plate (fig. 1) is moving with a velocity of 0.3 m/s. The viscosity of the fluid in between is 0.7×10^{-3} Pa.s.

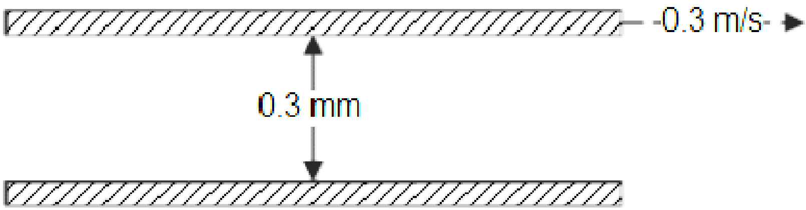

Fig. 1

5 + 7 = 12

3. (a) Explain the components of the molecular stress tensor. What is dilatational viscosity?

(b) Explain the term "collision cross section". Why the Chapman Enskog theory gives a better dependence of viscosity with temperature than the rigid sphere theory?

(3 + 2) + (2 + 5) = 12

Group – C

4. (a) Can we apply shell-momentum balance technique for deriving velocity profile of a fluid flowing inside an open channel when the flow is turbulent? – Justify your answer.

(b) A Newtonian fluid is in laminar flow in a narrow slit formed by two parallel walls at a distance 2B apart (Fig. 2). It is understood that B<< W, so that the edge effects are unimportant. Make a differential momentum balance and show that the velocity profile is given by $u_z = \left(\frac{P_0 - P_L}{2\mu L}\right) B^2 \left[1 - \left(\frac{x}{B}\right)^2\right]$

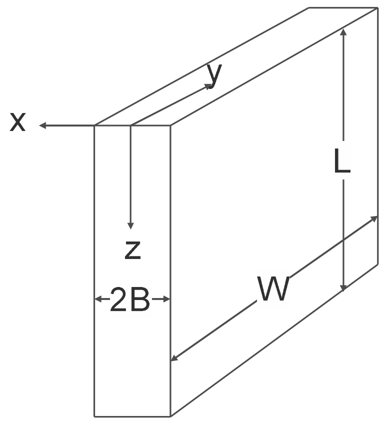

Fig. 2

4 + 8 = 12

5. The space between two co-axial vertical cylinders is filled with an incompressible fluid at constant temperature. The radii of the inner and outer wetted surfaces are κR and R respectively. The angular velocities of rotation of the inner and outer cylinders are ω_i and ω_o respectively. Determine the velocity distribution of the fluid within the annular portion. Assume the thickness of the cylinders is negligible.

12

Group – D

6. A copper wire has a radius 2 mm and a length of 5 m. For what voltage drop would be temperature rise at the wire axis be 10°C, if the surface temperature of the wire is 20°C? Derive the temperature profile using shell-energy balance to solve the problem. Given: Lorenz number for copper = 2.23 × 10^{-8} Volt²/K² and the current density is given by $I = k_e \frac{E}{L}$. k_e is the effective electrical conductivity, E is the voltage drop and L is the length of the wire.

12

7. (a) At a certain location in Hawaii, the ground temperature is 35°C. At 11 am, suddenly a volcanic eruption takes place and a pyroclastic flow at 200°C spreads over the ground for 4 hours upto 3 pm. The thermal conductivity of the soil is 0.5 W/m°C and thermal diffusivity is 0.15×10^{-6} m²/s. What will be the temperature at a distance of 10 m underneath the ground at 3 pm?

(b) For Pr<1, between thermal and momentum boundary layer, which one is thicker at a certain distance away from the leading edge? Justify your answer.

8 + 4 = 12

Group – E

8. (a) Predict the value of D_{AB} for the system CO (A) -CO_2 (B) at 296 K and 101.3 kPa total pressure.

Given:

$$D_{AB} = 0.0018583\sqrt{T^3\left(\frac{1}{M_A}+\frac{1}{M_B}\right)}\frac{1}{p\sigma_{AB}^2\Omega_{AB}}$$

p= pressure (atm); M: Molecular weight; σ_{AB} = Molecular distance (A°); Ω_{AB}: Collision integral

σ_A = 3.59 A°; σ_B = 3.996 A°; ε_A/k= 100 K; ε_B/k = 190 K

kT/ε_{AB}	0.9	0.95	1.0	1.05	1.1	1.15	1.20	1.25
Ω_{AB}	1.517	1.477	1.440	1.406	1.375	1.347	1.32	1.296

(b) Show that for a binary (A and B) mixture, the molecular flux of A through B is given as $J_A=N_A - x_A(N_A+N_B)$, where N is the molar flux and x denotes the mole fraction.

8 + 4 = 12

9. (a) Why the Chilton-Colburn analogy gives more accurate estimations of transport coefficients than Reynold's Analogy?

(b) A liquid A partially filling a test tube is evaporating through the stagnant film of air over it inside the test tube into the surrounding atmosphere where air is flowing with a high enough velocity to sweep away the vapours of A from the mouth of the test tube. Derive an expression for the concentration profile of A within the test tube.

6 + 6 = 12

Department & Section	Submission Link
CHE	https://classroom.google.com/c/MzEyNTIyMzc2NjU4/a/Mzc0NzI4MTg5Mzg3/details

Navier Stoke's Equation:

Cylindrical coordinates (r, θ, z):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{zz} = -\mu\left[2\frac{\partial v_z}{\partial z}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta z} = \tau_{z\theta} = -\mu\left[\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right]$$

$$\tau_{zr} = \tau_{rz} = -\mu\left[\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right]$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

Spherical coordinates (r, θ, ϕ):

$$\tau_{rr} = -\mu\left[2\frac{\partial v_r}{\partial r}\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{\theta\theta} = -\mu\left[2\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{\phi\phi} = -\mu\left[2\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right)\right] + (\tfrac{2}{3}\mu - \kappa)(\nabla \cdot \mathbf{v})$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left[r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right]$$

$$\tau_{\theta\phi} = \tau_{\phi\theta} = -\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right]$$

$$\tau_{\phi r} = \tau_{r\phi} = -\mu\left[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right]$$

in which

$$(\nabla \cdot \mathbf{v}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

Cartesian coordinates (x, y, z):

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Cylindrical coordinates (r, θ, z):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Spherical coordinates (r, θ, φ):

$$\frac{\partial \rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(\rho r^2 v_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\rho v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\rho v_\phi) = 0$$

Cartesian coordinates (x, y, z):[a]

$$\rho\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z}\right) = -\frac{\partial p}{\partial x} - \left[\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial y}\tau_{yx} + \frac{\partial}{\partial z}\tau_{zx}\right] + \rho g_x$$

$$\rho\left(\frac{\partial v_y}{\partial t} + v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z}\right) = -\frac{\partial p}{\partial y} - \left[\frac{\partial}{\partial x}\tau_{xy} + \frac{\partial}{\partial y}\tau_{yy} + \frac{\partial}{\partial z}\tau_{zy}\right] + \rho g_y$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial y}\tau_{yz} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Cylindrical coordinates (r, θ, z):[b]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta r} + \frac{\partial}{\partial z}\tau_{zr} - \frac{\tau_{\theta\theta}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta} - \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\theta}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta\theta} + \frac{\partial}{\partial z}\tau_{z\theta} + \frac{\tau_{\theta r} - \tau_{r\theta}}{r}\right] + \rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{1}{r}\frac{\partial}{\partial \theta}\tau_{\theta z} + \frac{\partial}{\partial z}\tau_{zz}\right] + \rho g_z$$

Spherical coordinates (r, θ, φ):[c]

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}\right) = -\frac{\partial p}{\partial r}$$
$$- \left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{rr}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta r}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi r} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}\right] + \rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r}\right) = -\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\theta}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\theta} + \frac{(\tau_{\theta r} - \tau_{r\theta}) - \tau_{\phi\phi}\cot\theta}{r}\right) + \rho g_\theta$$

$$\rho\left(\frac{\partial v_\phi}{\partial t} + v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_r + v_\theta v_\phi\cot\theta}{r}\right) = -\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}$$
$$- \left[\frac{1}{r^3}\frac{\partial}{\partial r}(r^3\tau_{r\phi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\phi}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\tau_{\phi\phi} + \frac{(\tau_{\phi r} - \tau_{r\phi}) + \tau_{\phi\theta}\cot\theta}{r}\right] + \rho g_\phi$$