# PHYSICS - I (PHYS 1001)

Time Allotted: 3 hrs

(i)

Full Marks: 70

Figures out of the right margin indicate full marks.

#### Candidates are required to answer Group A and <u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.

Candidates are required to give answer in their own words as far as practicable.

# Group – A (Multiple Choice Type Questions)

1. Choose the correct alternative for the following:

 $10 \times 1 = 10$ 

- The amplitude of a damped harmonic oscillator is:
  - (a) function of the initial conditions but independent of time
  - (b) function of the time but independent of initial conditions
  - (c) function of the initial conditions and time
  - (d) independent of the initial conditions and time.

(ii) A point with spherical polar coordinates  $\left(8, \frac{\pi}{3}, \frac{\pi}{6}\right)$  has Cartesian coordinates (a)  $\left(2, 2\sqrt{3}, 4\sqrt{3}\right)$  (b)  $\left(2, \sqrt{3}, 2\sqrt{3}\right)$ (c)  $\left(2\sqrt{3}, 2, 4\sqrt{3}\right)$  (d)  $\left(1, 2, 4\sqrt{3}\right)$ .

#### (iii) If *r* is the magnitude of the position vector $\vec{r}$ of a particle, then $\vec{\nabla} r^n =$ (a) $nr^{2-n}\hat{r}$ , $\hat{r}$ is unit vector (b) $nr^{n-2}\vec{r}$ , $\vec{r}$ is position vector (c) $nr^{n-2}\hat{r}$ , $\hat{r}$ is unit vector (d) 0

(iv) The central force  $\vec{F} = \frac{r(r-1)}{(r^2+1)}\hat{r}$  is attractive towards the origin O for (a) r > 1 (b) r < 0(c) 0 < r < 1 (d) all values of r

# (v) If the magnetic vector potential is given by $\vec{A} = (xy)\hat{i} + (yz)\hat{j} + (zx)\hat{k}$ , the magnetic field is: (a) function of x, y, z (b) function of x & y only (c) function of x & z only (d) function of y & z only.

(vi) The divergence of  $\vec{H}$ , in general, is (a)  $\vec{\nabla}.\vec{H} = 0$ (c)  $\vec{\nabla}.\vec{H} = \vec{\nabla}.\vec{M}$ where symbols have their usual meaning.

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(b)  $\vec{\nabla}.\vec{H} = -\vec{\nabla}.\vec{M}$ 

(d) None of the above.

- (vii) A wind is blowing from the equator towards the northern hemisphere. Due to earth's rotation, it will be deflected towards

   (a) east
   (b) west
   (c) north
   (d) south.
- (viii) For a forced harmonic oscillator system, the amplitude resonance condition can be achieved if the relation between the damping constant (*b*) and the natural frequency ( $\omega_0$ ) is:

(a) 
$$\omega_0 = b$$
 (b)  $\omega_0 = 2b$  (c)  $\omega_0 = \frac{b}{2}$  (d) All of the above.

- (ix) Laplace equation in electrostatics is given by (a)  $\nabla^2 \phi = \frac{\varepsilon_0}{\rho}$ (b)  $\nabla^2 \phi = -\frac{\rho}{\varepsilon_0}$ (c)  $\nabla^2 \phi = 0$ (d)  $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
- (x) To convert an elliptically polarized light into a circularly polarized light we need to use
  - (a) one quarter wave plate and one half wave plate
  - (b) two quarter wave plates
  - (c) two half wave plates
  - (d) one quarter wave plate.

#### Group - B

- 2. (a) Given the vector field  $\vec{A} = 5x^2 \sin\left(\frac{\pi x}{2}\right) \hat{i}$ , find divergence of  $\vec{A}$  at x = 1.
  - (b) Using vector identity evaluate  $\vec{\nabla} \cdot (\vec{A} \times \vec{r})$  when  $\vec{\nabla} \times \vec{A} = 0$ , where  $\vec{r}$  is the position vector and  $\vec{A}$  is any vector.
  - (c) Find the unit vector in cylindrical coordinate system, for the vector which is directed from the point ( $\rho$ ,  $\emptyset$ , 0) to a point z = h on Z axis.
  - (d) A vector  $\vec{A}$  is solenoidal and another vector  $\vec{B}$  is irrotational. What is the physical meaning of this statement?
  - (e) The height of a hill (in metres) is given by  $h(x, y) = 10(2xy 3x^2 4y^2 18x + 28y + 12)$ , where *y* and *x* are the distances (in metres) in the north and east directions from the origin *O*, respectively. How steep is the slope at a point 1m north and 1m east of the point *O*? In what direction is the slope steepest at that point?

2 + 2 + 2 + 3 + (2 + 1) = 12

- 3. (a) A particle moving in a central force describes a spiral trajectory  $r = e^{-\theta}$ . Show that the magnitude of the force is inversely proportional to  $r^3$ .
  - (b) The equation of the orbit of a particle moving under the influence of a central force is given by  $r = \frac{12}{3 + \cos \theta}$ . Justify the nature of the orbit and also find the semimajor axis of the orbit.

- (c) Using the operator relation  $\left(\frac{d}{dt}\right)_{fix} = \left(\frac{d}{dt}\right)_{rot} + \vec{\omega} \times ,$ 
  - (i) Establish the relation between  $\left(\frac{d^2\vec{r}}{dt^2}\right)_{fix}$  and  $\left(\frac{d^2\vec{r}}{dt^2}\right)_{rot}$  where  $\vec{r}$  is the position vector of a particle with respect to the rotating frame of reference, and *O* is

origin of both the fixed and rotating frames of references.

(ii) Prove that  $\left(\frac{d}{dt}\right)_{fix} |\vec{A}|^2 = \left(\frac{d}{dt}\right)_{fix} |\vec{A}|^2$ , where  $\vec{A}$  is any vector. 3 + (2 + 2) + (4 + 1) = 12

## Group – C

- 4. (a) An object of mass *m* is placed on a one dimensional frictionless horizontal table and attached to a massless spring of spring constant *k*. A resistive force  $(S_m v)$ acts on it where *v* is the velocity of the object and  $S_m$  is a positive constant. If the object is subjected to an external periodic driving force  $F = F_0 \sin(\omega_1 t)$  then
  - (i) Calculate the average power dissipated from the system by the resistive force and the average power delivered to the system by the external driving force.
  - (ii) Show that these two average powers are equal.
  - (b) Draw the state of polarization for the following light vector:

$$\vec{E} = 0.5\cos(\omega t - ky)\hat{\imath} + \cos\left(\omega t - ky + \frac{7\pi}{4}\right)\hat{k}$$

- (c) A particle is undergoing a damped vibration subject to the condition of initial displacement a and velocity *zero*. Find the trajectory of the particle in critical damping condition. (2 + 2 + 2) + 3 + 3 = 12
- 5. (a) The rate of loss of total energy of a damped harmonic oscillator is equal to the power dissipated by the damping force. Using the above statement, find out the differential equation of motion of the damped harmonic oscillator.
  - (b) Verify whether the function  $\Psi(x, t) = 5xt + 6e^{-t}$  represents a one dimensional classical wave or not. Further, comment on the wave velocity.
  - (c) Show that for a system in thermodynamic equilibrium at temperature *T*, the ratio of the probability of spontaneous emission to the probability of stimulated emission is given by  $e^{\frac{hv}{kT}} 1$ .
  - (d) What will be the change in the state of polarization of the emergent light from a half wave plate if the incident light is plane polarized? Give reason in support of your answer.

3 + 3 + 4 + (1 + 1) = 12

### Group – D

6. (a) The region between two concentric right circular cylinders contains a uniform charge density  $\rho$ . Use Poisson's equation to find the potential *V* in this region.

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- (b) Two concentric spherical shells of radii 0.1 m and 2.0 m are at V = 0 and V = 100 V respectively. Find electric field  $\vec{E}$  in free space between these concentric spherical shells. Also, obtain the surface charge density of the outer spherical shell.
- (c) A charge of  $500\pi$  *C* uniformly distributed over a circular ring of radius 5 m placed in YZ plane with its centre at origin. Write the expression for the electric field at (5,0,0) due to an infinitesimal charge *dQ* present at (5,  $\phi$ ) on the circular ring. Also obtain the total electric field at (5,0,0) due to the total charge distribution of the circular ring.

2 + (4 + 1) + (2 + 3) = 12

- 7. (a) A charge of 3 C is placed at (0,0,4) in front of a grounded infinite conducting plane, z = 0. Obtain the potential and electric field at (3,4,5) using image method. What is the value and position of the image charge? Explain why the image charge cannot be placed in z > 0 region.
  - (b) A charge is distributed along the *z* axis between  $z = \pm 5$  m with a uniform linear charge density  $\lambda = 20nC/m$ . Determine the electric field  $\vec{E}$  at (2,0,0) in Cartesian coordinates due to this charge distribution.
  - (c) If polarization  $\vec{P} = \chi \epsilon_0 \vec{E}$ , then express the electric displacement vector in terms of the electric field  $\vec{E}$ .
  - (d) Calculate the potential at  $(10, \pi/4)$  for two charges  $\pm 1C$  placed at (1,0,0) and (-1,0,0) where  $(r, \theta)$  is the polar coordinates, r is the distance from origin and  $\theta$  is the angle measured from the x axis in the anti clockwise sense.

(4 + 1) + 3 + 2 + 2 = 12

# Group – E

- 8. (a) In case of a linear magnetic media establish the relation  $\mu = \mu_0(1 + \chi_m)$ , where the symbols have their usual meaning.
  - (b) Show that the closed line integral of the magnetic field is independent of the integration path and depends only on the total amount of current enclosed by the integration path. Hence find out the value of the curl of a magnetic field.
  - (c) Find out the magnetic force per unit length between two infinitely long parallel straight wires at a distance d apart, carrying steady currents  $I_1$  and  $I_2$ . Assume the currents to be anti-parallel.
  - (d) A coil of *n* turns and area *A* rotates with a frequency  $\omega$  about a diameter that is perpendicular to a uniform magnetic field *B*. Calculate the peak emf induced in the coil.

3 + (2 + 2) + 2 + 3 = 12

- 9. (a) Using Biot-Savart law, obtain the expression of the magnetic field due to an infinitely long solenoid carrying a steady current, at an axial field point on one edge of the solenoid.
  - (b) Find out the magnetic dipole moment corresponding to an orbital electron rotating in a circular orbit of radius R with velocity v around a nucleus with atomic number Z.
  - (c) What type of magnetic material should we use as the core of armatures of dynamos and motors? Give reasons for your answer.
  - (d) Suppose that the magnetic field in some region has the form  $\vec{B} = c \ z \ \hat{i}$  where *c* is a positive constant. Find the force on a square loop (side *a*), lying in the *YZ* plane and centred at the origin, if it carries a current *I*, flowing counterclockwise, when we look down the *X* axis.

3 + 3 + (1 + 2) + 3 = 12

Department & Section	Submission Link
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CSBS	https://classroom.google.com/c/MzExNTcxOTg3Njcx/a/Mzc0MjMyODYwNjg4/details
CSE - A	https://classroom.google.com/c/MzExNTU4NjI2MzY1/a/Mzc0MjM4MjYwMzg4/details
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ECE - A	https://classroom.google.com/c/MzExNTQ5MjY3MzQ3/a/Mzc0MjMzMjA4MjM5/details
ECE - B	https://classroom.google.com/c/MzExNTgxMzQyMjcx/a/Mzc0MjMzNjI5OTQy/details
ECE - C	https://classroom.google.com/c/MzExNTgyMzA4MjI5/a/Mzc0MjMyODYwNjYx/details
IT	https://classroom.google.com/c/MzExNTU3MDg5Nzk2/a/Mzc0MjI5MjIyMDQz/details
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