DISCRETE MATHEMATICS
(CSEN 2102)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) A new company with just two employees, Sanchez and Patel, rents a floor of a building with 12 offices. The number of ways to assign different offices to these two employees is
(a) 132 (b) 144 (c) 121 (d) 23

(ii) A student can choose a computer project from one of three lists. The three lists contain 23, 15 and 19 possible projects, respectively. No project is more than one list. The number of possible projects to choose from is
(a) 6555 (b) 57 (c) 36 (d) 76.

(iii) The number of students that must be in a class to guarantee that at least two students receive the same score on the final exam. If the exam is graded on a scale from 0 to 100 points, is
(a) 100 (b) 50 (c) 102 (d) 110.

(iv) If $K_{n,m}$ is non-planner then
(a) $n=2, m=2$ (b) $n=2, m=3$ (c) $n=3, m=2$ (d) $n=4, m=4$

(v) Which one of the following graph has a perfect matching?
(a) P_5 (b) C_7 (c) K_5 (d) C_8

(vi) Let T be a tree having 50 vertices. Then $\chi(T) =$
(a) 3 (b) 4 (c) 2 (d) 1.

(vii) Let G^* be the graph dual to the graph G, then the number of edges of G is equal to
(a) number of edges of G^* (b) number of vertices of G^*
(c) number of regions determined by G^* (d) number of components by G^*

(viii) The number of solutions of $x_1 + x_2 + x_3 = 7$ where each $x_i \geq 0$, is
(a) $\frac{9!}{7!2!}$ (b) $\frac{9!}{7!}$ (c) $\frac{9!}{2!}$ (d) $\frac{9!}{3!}$

(ix) From the premises $p \to q$ and $q \to r$ one can conclude

(a) $p \vee \sim q$ (b) $\sim p \vee r$ (c) $p \wedge \sim r$ (d) $\sim p \vee \sim q$

(x) $\sim(\sim p \vee \sim q) \equiv$

(a) $\sim p \wedge \sim q$ (b) $p \vee q$ (c) $\sim p \vee \sim q$ (d) $p \wedge q$

Group – B

2. (a) Consider the following statements:

p: The flood destroys my house.

q: The fire destroys my house.

r: My insurance company will pay me.

Write down the following statements in p, q, r.

(i) "If the flood destroys my house or the fire destroys my house, then my insurance company will pay me".

(ii) "If my insurance company pays me, then the flood destroys my house or the fire destroys my house".

(iii) "If neither the flood destroy my house nor the fire destroy my house then my insurance company will not pay me".

(iv) "If my insurance company does not pay me, then the flood does not destroy my house and fire does not destroy my house".

Find which one of (ii), (iii) and (iv) is 1. Converse 2. Inverse 3. Contrapositive of the implication (i).

(b) Prove the following (without using truth table).

(i) $p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$.

(ii) $((p \vee \sim p) \to q) \to ((p \vee \sim p) \to r)) \to (q \to r)$ is a Tautology.

(4 + 2) + (3 + 3) = 12

3. (a) Find the Principal Disjunctive normal forms of the following:

(i) $(\sim p \to q) \wedge (q \leftrightarrow p)$.

(ii) $(p \wedge q) \vee (\sim p \wedge q) \vee (q \wedge r)$.

(b) Check the validity of the following arguments:

"If I pass B.Tech with high YGPA, I will be assured a good job. If I am assured of a good job then my father will be happy. My father is not happy. Therefore I do not pass B.Tech with high YGPA."

(3 + 3) + 6 = 12

Group – C

4. (a) Determine all integer solutions to the equation : $x_1 + x_2 + x_3 + x_4 = 7$, where $x_i \geq 0, \forall 1 \leq i \leq 4$.

(b) Prove the Principle of Inclusion-Exclusion for three sets A, B, C.

5 + 7 = 12

5. (a) Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$, where $n \geq 0$ and $F_0 = 0,\ F_1 = 1$.

(b) Prove that any subset of size 6 from the set $S = \{1,2,3.....,9\}$ must contain two element whose sum is 10.

(c) Find the coefficient of $x^a y^b z^c w^d$ in the expansion of $(x+y+z+w)^n$. [Here a,b,c,d are integers≥ 0, $n = a+b+c+d$].

5 + 3 + 4 = 12

Group – D

6. (a) Prove Konig's Theorem. The chromatic number of a simple graph having at least one edge is 2 if and only if the graph contains no circuits of odd length**.**

(b) State the definition of the chromatic polynomial of a simple graph. Prove that the chromatic polynomial of any simple graph is a polynomial.

6 + 6 = 12

7. (a) Prove that every simple connected planar graph has a vertex of degree at most 5.

(b) (i) Prove or disprove: Every sub-graph of a non-planar graph is non-planar.
(ii) Prove or disprove: Every proper sub-graph of $K_{3,3}$ is planar.

6 + (3 + 3) = 12

Group – E

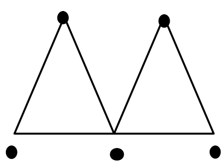
8. (a) State the definition of a chromatic number of a simple graph. Prove that chromatic number of a graph having several components equals the maximum of the chromatic number of its components.

(b) Prove that
(i) a perfect matching is a maximum matching;
(ii) a maximum matching is not necessarily a perfect matching;
(iii) a maximal matching is not necessarily a maximum matching.

6 + (2 + 2 + 2) = 12

9. (a) Find the number of perfect matching in $K_{8,8}$. Justify your answer and show your calculation in detail.

(b) Find the chromatic polynomial of the following graph. Justify your answer in detail.



6 + 6 = 12

Department & Section	Submission Link
CSE	https://classroom.google.com/c/MjIwOTEwODA4MjQ2/a/MjY1MTcxMzMwOTc0/details