MATHEMATICAL METHODS
(MATH 2001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) The value of the integral $\oint_C \frac{e}{z-2}\, dz$, where $C: |z-2| = 4$ is
(a) $2\pi i$ (b) $2\pi i e^2$ (c) πi (d) $\pi i\, e$.

(ii) If $f(z) = u(x,y) - iv(x,y)$ is analytic, then $f'(z)$ equals to
(a) $\frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$ (b) $\frac{\partial u}{\partial x} + i\frac{\partial u}{\partial y}$
(c) $\frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$ (d) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$.

(iii) If the function $f(z)$ is not analytic at $z = 0$; then $z = 0$ is called a
(a) ordinary point (b) singular point
(c) zero of $f(z)$ (d) none of these.

(iv) If $F(s)$ be the Fourier transform of $f(x)$, then the Fourier transform of $f(x-a)$ is
(a) $e^{is}F(s)$ (b) $F(s)$ (c) $e^{ias}F(s)$ (d) $e^{-ias}F(s)$.

(v) If $f(x) = x^2,\ -\pi \le x < \pi$ be expanded in Fourier Series as $\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ then b_1 is
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

(vi) The solution of the differential equation $(1-x^2)y'' - 2xy' + 12y = 0$ is
(a) $P_0(x)$ (b) $P_1(x)$ (c) $P_3(x)$ (d) $P_4(x)$

(vii) Let $P_n(x)$ be the Legendre Polynomial, then $P_2(x)$ is

(a) x (b) $\frac{1}{2}(3x^2 - 1)$

(c) 1 (d) $\frac{1}{3}(2x^2 - 1)$.

(viii) The solution of $xp + yq = z$ is

(a) $f(x, y)$ (b) $f\left(\frac{x}{y}, \frac{y}{z}\right)$

(c) $f(xy, yz)$ (d) $f(x^2, y^2)$

(ix) Equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \left(\frac{\partial z}{\partial y}\right)^2 = 0$ is of order

(a) 3 (b)2 (c) 1 (d) none of these.

(x) If the Fourier transform of $f(t)$ be $F(s)$, then the Fourier transform of $f(t)\cos at$ is

(a) $F(s + a)$ (b) $\frac{1}{2}\{F(s + a) - F(s - a)\}$

(c) $\frac{1}{2}\{F(s - a) - F(s + a)\}$ (d) $\frac{1}{2}\{F(s + a) + F(s - a)\}$.

Group – B

2. (a) Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin, although Cauchy-Riemann equations are satisfied.

(b) Prove that $u(x, y) = e^{-x}(x\sin y - y\cos y)$ is a harmonic function. Find $v(x, y)$ such that $f(z) = u + iv$ is analytic.

6 + 6 = 12

3. (a) Evaluate $\oint_C \frac{\cos^3 z}{\left(z-\frac{\pi}{4}\right)^3} dz$ where C is the circle $|z| = 1$.

(b) Find the residues of $f(z) = \frac{z}{(z-1)^4(z-2)(z-3)}$ at its pole and hence evaluate $\oint_C f(z)dz$, where C is the circle $|z| = 2.5$.

5 + 7 = 12

Group – C

4. (a) If $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$ prove that

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2-1}.$$

Hence show that $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots = \frac{1}{2}$.

(b) Obtain the Fourier series for the function $f(x)$ given by

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \leq x \leq 0 \\ 1 - \frac{2x}{\pi}, & 0 \leq x \leq \pi. \end{cases}$$

7+5= 12

5. (a) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} 1, & for\ 0 < x \leq \pi \\ 0, & for\ x > \pi. \end{cases}$$

Hence evaluate $\int_0^\infty \frac{1-c}{s} \sin sx\, ds$.

(b) Find the Fourier transform of $f(x) = e^{-a|x|}$, where $a > 0$. Hence find the Fourier transform of the function $f(x) = e^{-|x-3|}$.

6 + 6 = 12

Group – D

6. (a) Find the power series solution of the differential equation $2x^2y'' + xy' - (x+1)y = 0$ about $x = 0$.

(b) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$, where $J_n(x)$ is the Bessel's function of first kind.

7 +5= 12

7. (a) Prove the following recurrence relation for Bessel's function using generating function:

$$2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$$

where $J_n(x)$ is the Bessel's function of first kind.

(b) Show that $P_n(-x) = (-1)^n P_n(x)$. Hence deduce that $P_n(-1) = (-1)^n$, where $P_n(x)$ is the Legendre's polynomial of degree n.

6 + 6 = 12

Group – E

8. (a) Solve the following partial differential equation by Lagrange's method:

$$(y - z)p + (x - y)q = z - x.$$

(b) Solve: $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$.

6+6 = 12

9. (a) Solve the following one-dimensional heat equation by using Laplace transform method:

$$\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$$

with boundary conditions $u\left(\frac{\pi}{2}, t\right) = 0$, $\left(\frac{\partial u}{\partial x}\right)_{x=0} = 0$ and initial condition $u(x, 0) = 30\cos 5x$.

(b) Solve the following partial differential equation using suitable method:

$$z^2 = pqxy.$$

7+5 = 12

Department & Section	Submission Link
AEIE/CE/ECE/EE	https://classroom.google.com/c/MjE4MTgwOTgzNTY0/a/Mjc0NTMyMjc2OTk3/details