MATHEMATICAL AND STATISTICAL METHODS
(MATH 2101)

Time Allotted : 3 hrs   **Full Marks : 70**

Figures out of the right margin indicate full marks.

*Candidates are required to answer Group A and
<u>any 5 (five)</u> from Group B to E, taking <u>at least one</u> from each group.*

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

    (i)   The mean and standard deviation of a standard normal distribution are
          (a) $1, 0$ (b) $0, 1$ (c) $1, 1$ (d) $0, 0$

    (ii) The harmonic mean (H.M) of $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ is
          (a) $\frac{25}{12}$ (b) $\frac{48}{25}$ (c) $\frac{25}{48}$ (d) $\frac{2}{5}$

    (iii) The period of the function $f(x) = \cos(\sqrt{3}\, x)$ is
          (a) 2π (b) $\frac{\pi}{\sqrt{3}}$ (c) $\frac{2\pi}{3}$ (d) $\frac{2\pi}{\sqrt{3}}$

    (iv) The function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x < \pi \end{cases}$ is symmetric about
          (a) x-axis   (b) y-axis   (c) origin   (d) both x and y-axes.

    (v)   The value of a_0 in the Fourier series expansion of $f(x) = \sin 3x$ in $(-\pi, \pi)$ is
          (a) $\frac{1}{2}$ (b) $-\frac{2}{3}$ (c) 0 (d) $\frac{2}{3}$

    (vi) The divided difference $[x_0, x_1, x_2] =$
          (a) $\frac{[x_1,x_2]-[x_0,x_1]}{x_2-x_1}$ (b) $\frac{[x_1,x_2]-[x_0,x_1]}{x_1-x_2}$
          (c) $\frac{[x_0,x_2]-[x_0,x_1]}{x_2-x_1}$ (d) $\frac{[x_1,x_2]-[x_0,x_1]}{x_2-x_0}$

    (vii) The solution of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$ is
          (a) $z = \varphi_1(y + x) + x\varphi_2(y - x)$ (b) $z = \varphi_1(y + x) + \varphi_2(y - x)$
          (c) $z = \varphi_1(y + x) + x\varphi_2(y + x)$ (d) $z = \varphi_1(y^2 - x^2)$.
          where φ_1 and φ_2 are arbitrary functions.

(viii) The degree and order of the partial differential equation $r^2 + t^2 + 2s = 0$ is
(where $= \frac{\partial^2 z}{\partial x^2}, t = \frac{\partial^2 z}{\partial y^2}, s = \frac{\partial^2 z}{\partial x \partial y}$.)

    (a) 2,2   (b) 2,1   (c) 1,2   (d) 1,1

(ix) Which of these following statements is **NOT** true?
    (a) $E^{-n} f(x) = f(x - nh)$ (b) $\Delta = E - 1$
    (c) $E^{-1} = 1 + \nabla$ (d) $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

(x) If $f(x)$ be a polynomial of degree 3 in x then,
    (a) $\Delta^4 f(x) = 0$ (b) $\Delta^4 f(x) = 3$
    (c) $\Delta^3 f(x) = x$ (d) $\Delta^3 f(x) = 0$ *always*.

Group – B

2. (a) Obtain the partial differential equation by eliminating arbitrary functions from the relation $z = yf(x) + xg(y)$.

(b) Solve: $p + 3q = 5z + tan(y - 3x)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

                                                     6 + 6 = 12

3. (a) Using Charpit's method find the complete integral of the equation $p^2 x + qy = z$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$.

(b) Solve the second order homogeneous linear partial differential equation $D^2 + 3DD' + 2D'^2 = x + y$ where $\equiv \frac{\partial}{\partial x} D' \equiv \frac{\partial}{\partial y}$.

                                                        6 + 6 = 12

Group – C

4. (a) Using Lagrange's interpolation formula calculate the value of y when $x = 102$ from the following table.

x: 93.0   96.2   100.0   104.2   108.7
y: 11.38   12.80 14.70   17.07   19.91.

(b) Using Newton's forward Interpolation formula, compute $log_{10} 2.15$ from the following table.

x:	2.0	2.2	2.4	2.6	2.8	3.0
$log_{10} x$:	0.30103	0.34242	0.38021	0.41497	0.44716	0.47721

                                                        6 + 6 = 12

5. (a) Using Newton's divided difference formula, find $f(x)$ as a polynomial in x from the following table.

x:	-1	0	1	2	3	4
y:	-16	-7	-4	-1	8	29.

(b)   Evaluate the integral $\int_0^1 (4x - 3x^2)\,dx$ approximately using trapezoidal rule by taking $n = 10$.

6 + 6 = 12

Group – D

6. (a)   Obtain the Fourier series for the periodic functions $f(x) = e^{-x}$ in $-\pi < x < \pi$ where $f(x + 2\pi) = f(x)$.

(b)   Find the Fourier cosine series of the function $f(x) = \begin{cases} x^2, & 0 \le x < 2 \\ 4, & 2 \le x \le 4 \end{cases}$.

6 + 6 = 12

7. (a)   Write the sine series expansion of $f(x) = \begin{cases} 1, & 0 < x \le \frac{\pi}{2} \\ 2, & \frac{\pi}{2} < x < \pi \end{cases}$ in $[0, \pi]$. Discuss its convergence at $x = \frac{\pi}{2}$.

(b)   Using the Parseval's Identity for the Fourier series of $f(x) = \cos\left(\frac{x}{2}\right)$ in $(-\pi, \pi)$, prove that $\sum_{n=1}^{\infty} \frac{1}{(4n^2-1)^2} = \frac{\pi^2-8}{16}$.

6 + 6 = 12

Group – E

8. (a)   If X has uniform distribution with parameter a, b then find (i) $E(X)$, (ii) $Var(X)$.

(b)   The distribution of number of road accidents per day in a city is a poisson distribution with mean 4. Find the approximate number of days out of hundred days when there will be
(i)   no accidents,
(ii) at least two accidents,
(iii) between 2 and 5 accidents (both inclusive).

6 + 6 = 12

9. (a)   Calculate the median and mode of the following frequency distribution:

Height in inches:	56 − 60	61 − 65	66 − 70	71 − 75	76 − 80
No. of persons:	7	25	43	28	7

(b)   Out of two regression lines given by $x + 2y = 5$ and $2x + 3y = 8$, which one is regression line of x on y? Find the mean of x and y. Also find the correlation coefficient between x and y.

6 + 6 = 12